Conservative Linear Line Flow Constraints for AC Optimal Power Flow

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1 Introduction

The AC optimal power flow (OPF) problem is non-convex and difficult to solve quickly and reliably for large-scale grids. Hence, various linear approximations are used, which may lead to solutions that are AC infeasible. To reduce the problem complexity while preserving feasibility, we linearize only line flow inequality constraints in the AC OPF problem. We investigate the shape of these nonlinear constraints and present methods for producing their conservative linear approximations.

2 Background and Goals

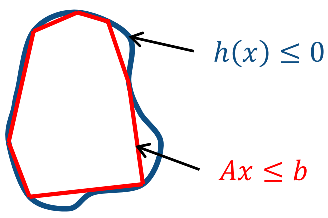
Problem Formulation
AC OPF is a non-convex large-scale optimization problem of the form:

$\underset{x}{\text{minimize}}$	$f(x)$	Cost (usually convex)
subject to	$g(x) = 0$	Kirchhoff's laws (nonlinear)
	$h(x) \leq 0$	Thermal limits (non-convex)
	$x^l \leq x \leq x^u$	

Line flow constraints such as thermal limits are non-convex inequality constraints. NLP solvers handle linear inequalities more efficiently.

Goal
Approximate $h(x) \leq 0$ with $Ax \leq b$.
This will reduce the complexity of the optimization problem.

Find A and b s.t. the approximation is:
- Conservative ($Ax \leq b \rightarrow h(x) \leq 0$)
- High-quality



3 Shape of Line Flow Constraints

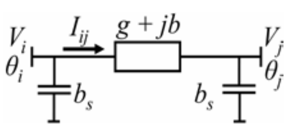
In polar coordinates, the magnitude of current flowing in a line is given by:

$$I_{ij}(\underbrace{V_i, V_j, \theta_{ij}}_{x}) = \alpha_1 V_i^2 + \alpha_2 V_j^2 + V_i V_j (\alpha_3 \cos\theta_{ij} + \alpha_4 \sin\theta_{ij}),$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ depend on g, b, b_s.

Thermal limit is of the form

$I_{ij}(V_i, V_j, \theta_{ij}) \leq I_{ij}^{\max}$ (1)



Observation: the shape of feasible region of (1) depends on the line type

Line is between	Feasible region		Boundary
PV buses	$h(x) \leq 0$	convex	linear
PV bus and PQ bus	$h(x) \leq 0$	convex*	nonlinear
PQ buses	$h(x) \leq 0$	non-convex**	nonlinear

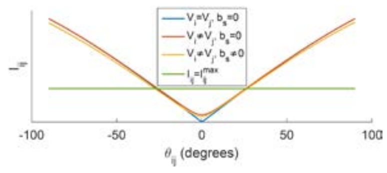
*The region may be non-convex but then (1) is never binding since $|\theta_{ij}| > \pi/3$
**For realistic lines, the non-convexities are small

Conclusion: constraint (1) can be approximated by linear inequalities

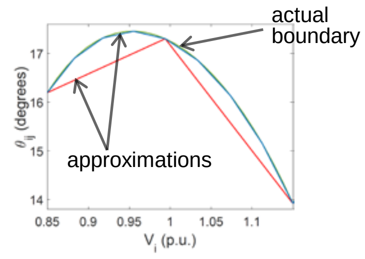
4 Approximations to Line Flow Constraints

The algorithm for obtaining conservative linear approximations $Ax \leq b$ and the quality of these approximations depend on the line type.

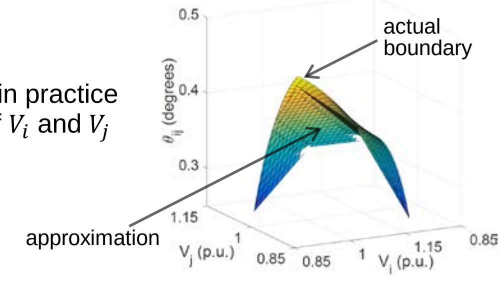
Line between PV buses:
- approximation error is zero
- A and b are obtained analytically



Line between PV bus and PQ bus:
- approximation error can be made arbitrarily small
- A and b are obtained using bisection algorithm



Line between PQ buses:
- approximation error is small in practice and depends on the range of V_i and V_j
- A and b are obtained using bisection algorithm



4 AC OPF with Linear Line Flow Constraints

AC OPF with linear line flow constraints can be solved faster yet the solution is feasible and the difference in the cost function is small

System	Type of constraints	Number of constraints	ΔCost (%)	Time (s)	
				SNOPT	KNITRO
118 bus	nonlinear	186	-	1.26	28.61
	linear (5%*)	718	0.0940	0.17	0.63
	linear (0.5%)	1460	0.0074	0.22	0.83
1354 bus	nonlinear	1991	-	488.68	DNC
	linear (5%)	17142	0.0551	7.79	848.97
	linear (0.5%)	34630	0.0084	16.66	883.38

*inaccuracy of linear approximation

5 Conclusion

The following conclusions can be drawn:
- The shape of feasible region of the line flow constraint depends on the type of the nodes that the line is connected to
- Nonlinear line flow constraint can be approximated with high accuracy by a set of linear inequality constraints, which can be easily determined
- NLP solvers obtain a solution to the AC OPF problem with linear line flow constraints faster despite a larger problem size

6 References

1. P. Lipka, R. O'Neill, and S. Oren, "Developing line current magnitude constraints for IEEE test problems", Technical Report, FERC, 2013.
2. C. Coffrin, P. Van Hentenryck, and R. Bent, "Approximating line losses and apparent power in AC power flow linearizations", in 2012 IEEE Power and Energy Society General Meeting, July 2012, pp. 1–8.