Optimization under Uncertainty
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1 A Similarity-Based Approach (Buhmann et al., 2013)

**Classical optimization**: Select a solution \( s^* \) from a set of possible solutions \( S \) (e.g., st-paths, matchings, ...) that minimizes an objective function \( f : S \rightarrow \mathbb{Q}_0^+ \)

**Situation here**: Problem generator \( \mathcal{P}\mathcal{S} \) generates instances \( I : S \rightarrow \mathbb{Q}_0^+ \) that differ due to noise; no knowledge about \( \mathcal{P}\mathcal{S} \) or the type of the noise

**Goal**: Given \( I_1, I_2 \) generated by \( \mathcal{P}\mathcal{S} \), find a solution that is likely to be good for other instances from \( \mathcal{P}\mathcal{S} \)

**Approach**: For each instance \( I \), consider its \( \rho \)-approximation set

\[
A_\rho(I) = \{ s \in S | I(s) \leq \rho \cdot \min_{s \in S} I(s) \}
\]

- For a well-tuned parameter \( \rho^* \), select a solution from \( A_\rho(I_1) \cap A_\rho(I_2) \) uniformly at random

- Finding the “right” \( \rho^* \) is nontrivial \( \sim \) Use \( \rho^* \) that maximizes the Similarity of \( I_1 \) and \( I_2 \),

\[
\frac{|A_\rho(I_1) \cap A_\rho(I_2)|}{|A_\rho(I_1)||A_\rho(I_2)|}
\]

2 Robust Routing in Urban Public Transportation

**Given**: Public transportation network with a planned timetable, “observed timetables” of past few days, origin \( s \), destination \( t \), latest allowed arrival time \( t_A \)

**Goal**: Recommend an st-journey that is robust against (observed) “typical” delays, i.e., that likely reaches \( t \) at time \( t_A \) at the latest.

**Solution concept**: Journeys. Depart from \( s \) at \( t_D \), use the sequence of lines \( l_1(\text{bus}), l_2(\text{bus}), \ldots, l_k(\text{bus}) \), change line at transfer stops \( s_1, s_2, \ldots, s_k-1 \).

**Observation**: Learning based methods can account for typical delays.

3 Robust Routing in Road Networks

**Goal**: Computing routes according to robust criteria (e.g., maximum similarity, first intersection)

**Setting**: Time-dependent FIFO road networks

**Issues**: Polynomial-time algorithms are not likely to exist

**Solution**: Heuristic techniques with good practical performance (e.g., bi-directional search, pruning, ...)

4 Geometric Uncertainty

**Classical setting**: Compute geometric objects (e.g., spanning trees) of a point set

**Imprecise points**: The location of each point is known to be located in an own occurrence region. Multiple realizations are possible:

**Question**: For which realization is the object as good/bad as possible?

**Objects studied**: Minimum weight spanning trees, minimum diameter spanning trees, shortest paths in geometric graphs

5 Comparison Errors – How Well Can We Sort?

**Model**: Set of elements with a strict linear order. An (unknown) upper bound \( k \) of wrong comparisons. Errors are recurring. Perform each comparison once.

**Approach 1**: Find a permutation with the least number of contradictions.

**Approach 2**: Sort the elements by the number of “lost” comparisons.

<table>
<thead>
<tr>
<th>Measures of Unsortedness</th>
<th>APPROACH 1</th>
<th>APPROACH 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INVERSIONS</strong></td>
<td>( 2k )</td>
<td>&lt; ( 4k )</td>
</tr>
<tr>
<td><strong>TOTAL DISLOCATION</strong></td>
<td>( 4k )</td>
<td>( k + 1 )</td>
</tr>
<tr>
<td><strong>MAX DISLOCATION</strong></td>
<td>( 2k )</td>
<td>( k + 1 )</td>
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