ETH zürich

Algorithms as Mechanisms: The Price of Anarchy of Relax-and-Round

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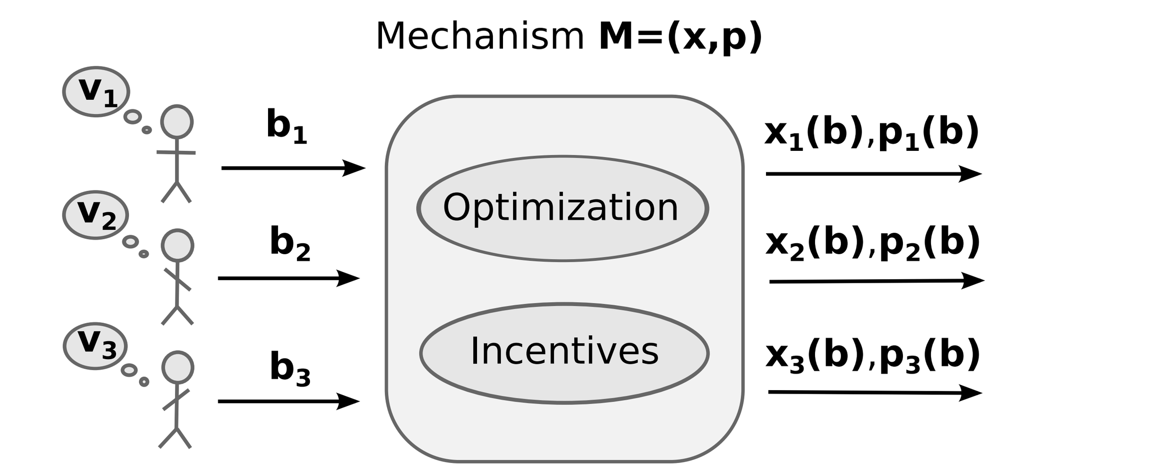
Motivation

The Dilemma

In mechanism design we want to optimize some objective, but the data is provided by strategic agents



Utility $u_i = v_i(x_i) - p_i$ Social welfare $\sum_{i \in N} v_i(x_i)$

One Way Out

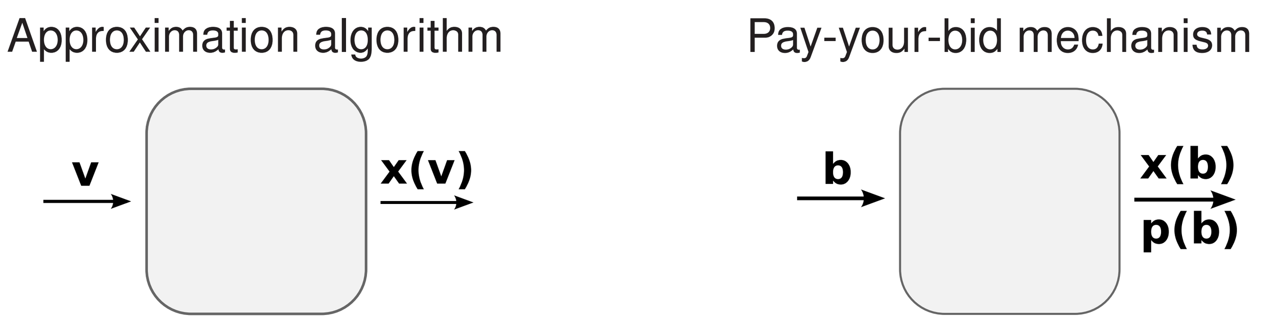
Truthful mechanisms incentivize agents to reveal their data truthfully

Black-Box Reductions: Lavi, Swamy [FOCS'05], Briest, Krysta, Vöcking [STOC'05], Dughmi and Roughgarden [FOCS'10], and Dughmi, Roughgarden, Yan [STOC'11]

Drawbacks that we avoid: Complicated mechanisms, natural barriers

Our Approach

Take an approximation algorithm and combine it with a simple pay-ment rule such as pay-your-bid



where $p_i(b) = b_i(x_i(b))$

Approximation ratio

$$\max_v \frac{\sum_i v_i(OPT(v))}{\mathbf{E}[\sum_i v_i(ALG(v))]}$$

Price of Anarchy

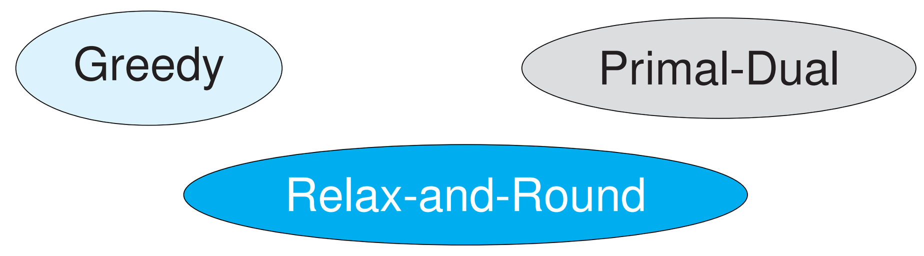
$$\max_v \max_{b \in NE(v)} \frac{\sum_i v_i(OPT(v))}{\mathbf{E}[\sum_i v_i(ALG(b))]}$$

Question: For which algorithms does this lead to mechanisms with near-optimal equilibria?

Main Result

Class of Algorithms with Near-Optimal Equilibria

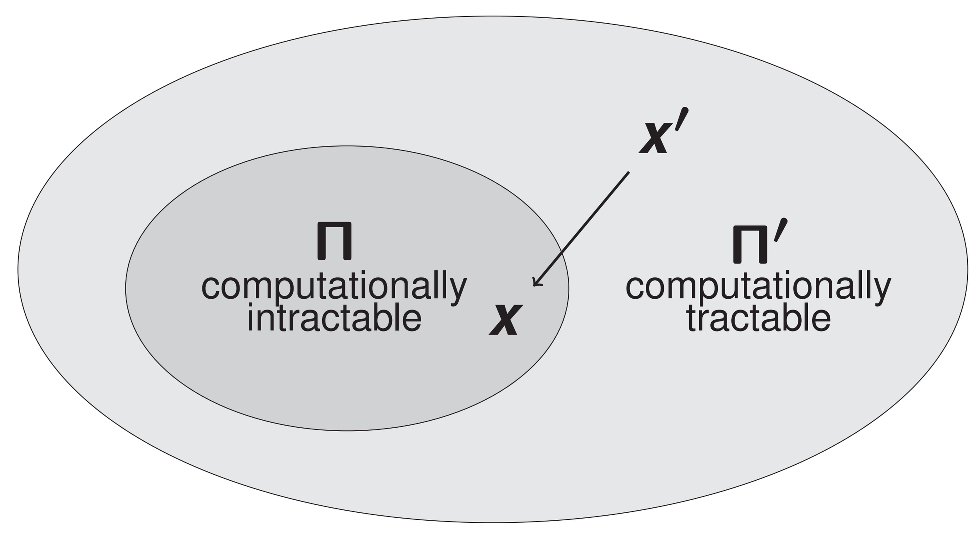
We show that relax-and-round algorithms translate approximation guarantees into Price of Anarchy guarantees



Lucier and Borodin [SODA'11]: Greedy with approximation ratio $\alpha \Rightarrow$ Price of Anarchy of $O(\alpha)$

Our result: Also true for relax-and-round if rounding is oblivious and relaxation is smooth

Relax-and-Round and Oblivious Rounding



LP-based relaxation
SDP-based relaxation
⋮

α-approximate oblivious rounding: For all w, $\mathbf{E}[w(x)] \geq \frac{1}{\alpha} w(x')$
i.e., rounding does **not** require knowledge of the objective function

Formal Statement of Main Result

Theorem

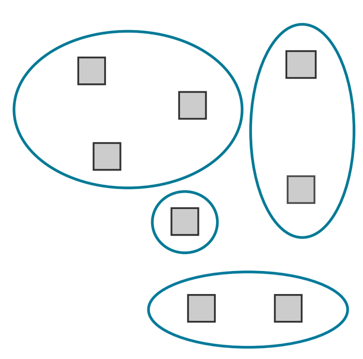
If pay-your-bid mechanism M is derived from pay-your-bid mechanism M' via α-approximate oblivious rounding and M' is (λ, μ)-smooth, then M is $(\lambda/(2\alpha), \mu)$-smooth

Corollary

Price of Anarchy $\leq \beta$ via smoothness for M' $\Rightarrow$ Price of Anarchy $\leq 2\alpha\beta$ via smoothness for M

Applications

Combinatorial Auctions

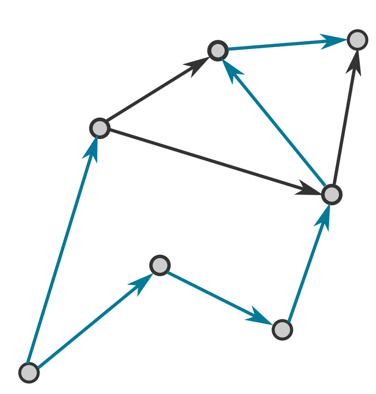


Input: Players N, items M, $\mathcal{MPH}-k$ valuations $v: 2^M \to \mathbb{R}_{\geq 0}$, where $v_i(S) = \max_\ell \sum_{T \subseteq S, |T| \leq k} v^\ell_{i,T}$
Output: Allocation of bundles of items to players

Theorem

There is a pay-your-bid mechanism that is based on oblivious round-ing and achieves a Price of Anarchy of at most $4\frac{e}{e-1}$ for XOS valua-tions and at most $O(k^2)$ for $\mathcal{MPH}-k$ valuations.

Single-Source Unsplittable Flow

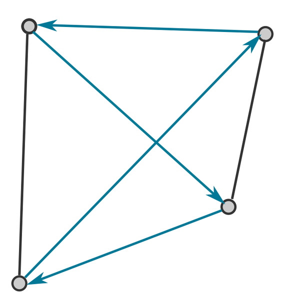


Input: Graph $G = (V, E)$ with capacities $(c_e)_{e \in E}$, source node s, target nodes $t_1, \ldots, t_n$, demands $d_1, \ldots, d_n$, and values $v_1, \ldots, v_n$
Output: Routing of flows that respects capacities

Theorem

If the minimum edge capacity is by a logarithmic factor larger than the maximum demand, then there is an oblivious rounding-based, pay-your-bid mechanism with a Price of Anarchy of at most $2(1+\epsilon)$.

Maximum Traveling Salesman



Input: Complete digraph $G = (V, E)$ with weights $(w_e)_{e \in E}$
Output: Hamiltonian cycle C that maximizes $\sum_{e \in C} w_e$

Theorem

There is an oblivious rounding-based, pay-your-bid mechanism for max-TSP with a Price of Anarchy of at most 9.

Sparse Packing Integer Programs

$$\begin{aligned} \max \quad & \textstyle\sum_{i \in N} v_i(x) \\ s.b. \quad & Ax \leq c, \\ & \textstyle\sum_{k \in K} x_{i,k} \leq 1 \ \forall i \in N, \\ & x_{i,k} \in \{0,1\} \ \forall i \in N, \forall k \in K \end{aligned}$$

Column sparsity:
$d = \max_{j,k} |S_{j,k}|$, where $S_{j,k} = \{\ell \mid A_{\ell,j,k} \neq 0\}$

Theorem

There is an oblivious rounding-based, pay-your-bid mechanism with a Price of Anarchy of at most at most $16d(d+1)$.