

Formal Methods and Functional Programming

Model Checking

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LTL Model Checking Problem

Given a finite transition system TS and an LTL formula ϕ ,
decide whether $t \models \phi$ for all $t \in \mathcal{T}(TS)$

- We need to check inclusion of traces
 - LTL formula ϕ describes a set of traces $P(\phi)$
 - We need to determine whether or not $\mathcal{T}(TS) \subseteq P(\phi)$
- Naïvely searching all traces is not an option (infinite length)

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Peter Müller—Formal Methods and Functional Programming, SS14 p. 295

Checking Regular Safety Properties

- A safety property is regular if its bad prefixes are described by a regular language over the alphabet $\mathcal{P}(AP)$
- Every invariant over AP is a regular safety property
 - For the property defined by $\Box p$, all bad prefixes start with S^*T where S describes any subset of $\mathcal{P}(AP)$ that contains p , and T any subset that does not contain p
 - For example, bad prefixes for $\Box open$ are described by $(\{open\} \mid \{open, closed\})^*(\{\} \mid \{closed\})$
- Non-regular safety properties also exist
 - Vending machine: at least as many coins inserted as drinks dispensed
 - Bad prefixes: regular languages “cannot count”

Checking Regular Safety Properties: Approach

- Look at all finite prefixes $\mathcal{T}_{fin}(TS)$ of the traces $\mathcal{T}(TS)$ of a transition system TS
- Check whether $\mathcal{T}_{fin}(TS)$ contains a bad prefix

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2. Describe bad prefixes of regular safety property P by finite automaton $\mathcal{FA}_{\bar{P}}$
3. Construct finite automaton for product of \mathcal{FA}_{TS} and $\mathcal{FA}_{\bar{P}}$

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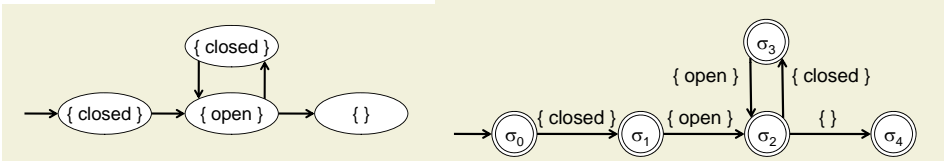
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3. Construct finite automaton for product of \mathcal{FA}_{TS} and $\mathcal{FA}_{\bar{P}}$
4. Check if the resulting automaton has any reachable accepting states
 - If not, the property P is never violated by computations of TS
 - If yes, the property P is violated
Each word in the accepted language of the product automaton is a counterexample
(i.e., a bad prefix of P that is a prefix of a computation of TS)

Reminder: Finite Automata

- A finite automaton (FA) is a tuple $(Q, \Sigma, Q, \delta, q_0, F)$
 - Q : a finite set of states
 - Σ : a finite alphabet
 - δ : a transition relation, $\delta \subseteq Q \times \Sigma \times Q$
 - q_0 : an initial state
 - $F \subseteq Q$: a set of accepting states

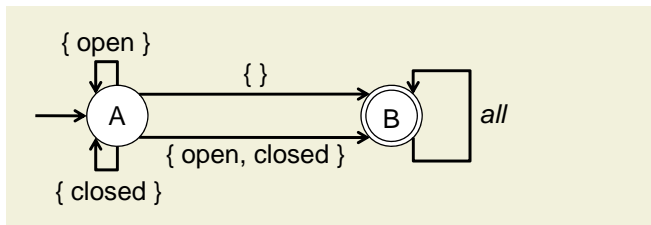
Step 1: Finite Automaton for Finite Prefixes

- Given a transition system $TS = (\Gamma, \sigma_I, \rightarrow)$, propositions AP , and labeling function L
- The automaton $\mathcal{FA}_{TS} = (Q, \Sigma, \delta, q_0, F)$ accepts $\mathcal{T}_{fin}(TS)$
 - $Q = \Gamma \cup \{\sigma_0\}$, where $\sigma_0 \notin \Gamma$
 - $\Sigma = \mathcal{P}(AP)$
 - $\delta = \{(\sigma, p, \sigma') \mid \sigma \rightarrow \sigma' \text{ and } p \in L(\sigma')\} \cup \{(\sigma_0, p, \sigma_I) \mid p \in L(\sigma_I)\}$
 - $q_0 = \sigma_0$
 - $F = Q$ (accept any prefix of a computation)
- Example: `o:=o+1; while * do o:=o-1; o:=o+1 end; o:=o+1`



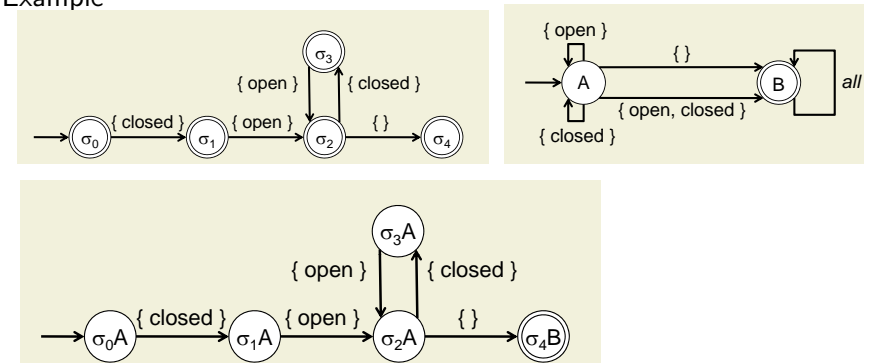
Step 2: Finite Automaton for Bad Prefixes

- By definition, bad prefixes are described by a regular language
- Apply standard construction to obtain FA $\mathcal{FA}_{\bar{P}}$ from regular expression
- Example: $\Box((open \vee closed) \wedge \neq (open \wedge closed))$
 - Bad prefixes start with $(\{open\} \mid \{closed\})^*(\{\} \mid \{open, closed\})$



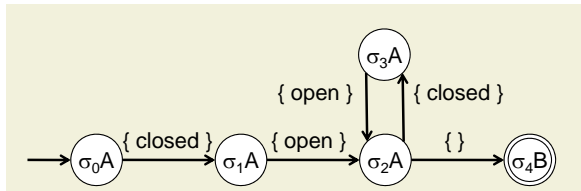
Step 3: Finite Automaton for Product

- Construct FA $\mathcal{FA}_{TS \cap \bar{P}}$ that accepts the intersection of the languages accepted by \mathcal{FA}_{TS} and $\mathcal{FA}_{\bar{P}}$
- Apply standard construction for product of two FA
- Example



Step 4: Check Emptiness

- If $\mathcal{FA}_{TS \cap \bar{P}}$ accepts a word w then
 - $w \in \mathcal{T}_{fin}(TS)$ because it is accepted by \mathcal{FA}_{TS} and
 - w is a bad prefix because it is accepted by $\mathcal{FA}_{\bar{P}}$
 - Therefore, P is not satisfied, and w is a counterexample
- Apply standard algorithm to check emptiness of FA
- Example



- Accepts $\{closed\}\{open\}(\{closed\} \mid \{open\})^*\{\}$
- Smallest counterexample: $\{closed\}\{open\}\{\}$
- Counterexample can be mapped back to transition system

LTL Model Checking: Approach

1. Describe traces $\mathcal{T}(TS)$ by Büchi automaton \mathcal{BA}_{TS}
 - Construction is analogous to \mathcal{FA}_{TS}

Büchi Automata

- Büchi automata are similar to finite automata, but accept infinite words
- A Büchi automaton (BA) is a tuple $(Q, \Sigma, Q, \delta, q_0, F)$
 - Q : a finite set of states
 - Σ : a finite alphabet
 - δ : a transition relation, $\delta \subseteq Q \times \Sigma \times Q$
 - q_0 : an initial state
 - $F \subseteq Q$: a set of accepting states
- A run of a BA accepts its input if it passes infinitely often through an accepting state
- Büchi automata enjoy many of the properties of finite automata
 - We can construct the product of two BA
 - Emptiness is decidable

LTL Model Checking: Approach

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 - We omit the details here

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Complexity Results

For a finite transition system TS and an LTL formula ϕ , the model checking problem $TS \models \phi$ is solvable in $\mathcal{O}(|TS| \times 2^{|\phi|})$

- $|TS|$ is the size of the transition system (which grows exponentially in the number of variables, processes, and channels)
- $|\phi|$ is the size of ϕ ; exponential complexity comes from the construction of $\mathcal{BA}_{\neg\phi}$

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 - We omit the details here
3. Construct BA for product of \mathcal{BA}_{TS} and $\mathcal{BA}_{\neg\phi}$
4. Check whether intersection is empty
 - If intersection is non-empty, property ϕ is violated
 - Each word in the intersection is a counterexample

Advanced Model Checking Techniques

- On-the-fly model checking
 - Often violation of a property can be detected without checking all possible states or traces (for instance, $\Box p$)
 - Generate transition system and check property step-by-step
 - Implemented in Spin
- Partial order reduction
 - Remove redundancy from different interleavings of concurrent executions
 - Code segments that operate only on local state are not affected by interleaving
 - Implemented in Spin

Advanced Model Checking Techniques (cont'd)

- Bounded model checking
 - Check only prefixes of traces up to a certain length
 - Closer to testing than verification
 - Very effective in practice
- Symbolic model checking
 - Uses sets of states rather than individual states
 - Sets of states are represented through boolean functions
 - Very efficient data structure: binary decision diagram (BDDs)
 - Typically used to check branching-time properties
 - Can deal with larger models

Conclusions

- Variety of approaches
 - Best method depends on application area
- Tool support is essential
 - Proofs are tedious and error-prone
 - Some tools have reached maturity for industrial applications