

Conscious and Unconscious Strategy Discoveries: A Microgenetic Analysis

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We examined second graders' strategy discoveries on a task that could be solved either by arithmetic computation or by a simple insight. Strategy use was assessed on each trial through both an implicit measure and an explicit one. The results provided unambiguous evidence that strategies can be discovered without conscious awareness. Almost 90% of children showed the insight at an implicit, unreportable level before they showed it at an explicit, reportable one. When children received the relevant problems on each trial, 80% reported the insight within 5 trials of when their solution times indicated that they first generated it. Implications for understanding the relation between unconscious and conscious discoveries and for the role of competing activations in strategy discovery are discussed.

Understanding changes in children's thinking poses a variety of challenges. Some of the challenges are conceptual. Describing and explaining cognitive–developmental change presents all of the difficulties of describing and explaining children's thinking at any given time, and it adds difficulties of understanding what is changing and how the changes are being achieved. Other challenges are methodological. Examining children's thinking at intervals of 1 to 3 years, as is done in most cross-sectional and longitudinal studies, indicates what changes occurred in the period, but it is insufficient to indicate how the changes occurred.

Microgenetic methods provide a means for obtaining the types of detailed data on changing competence that are needed to understand cognitive growth. Siegler and Crowley (1991) defined such methods in terms of three properties: (a) Observations span as large a portion as possible of the period in which rapid change in the competence occurs; (b) the density of observations within this period is high relative to the rate of change in the phenomenon; and (c) observations are examined on an intensive trial-by-trial basis, with the goal of understanding in detail the process of change.

The high density of observations is especially important for understanding rapidly changing competence. It provides the data needed to determine what actually goes on during the changes, rather than limiting understanding to what can be inferred from comparing performance before and after the change. For example, it allows identification of the exact trial on which a child first uses a new strategy, which in turn allows analysis of what led to the discovery and how it was

generalized beyond its initial context. Such dense sampling of changing competence also allows identification of transitional strategies that may be short lived but crucial for ushering in more enduring approaches. Microgenetic methods have proved useful for understanding changes in children's thinking in such diverse areas as scientific reasoning (Kuhn, Amsel, & O'Laughlin, 1988; Kuhn, Garcia-Mila, Zohar, & Andersen, 1995; Schauble, 1990, 1996), memory strategies (Bray, Saarnio, Borges, & Hawk, 1994; Coyle & Bjorklund, 1997), understanding of devices (Granott, 1993), map drawing and language development (Karmiloff-Smith, 1992), and simple addition (Siegler & Jenkins, 1989).

The purpose of the present study was to test whether the microgenetic method could also advance understanding of the role of consciousness in children's strategy discoveries. The task involved an arithmetic insight problem.

The Inversion Problem

Inversion is the principle that adding and subtracting the same number leaves the result unchanged. Knowing this principle allows solution of problems such as $28 + 36 - 36$ through use of a *shortcut strategy*: Simply ignore the number that was both added and subtracted. Generation of this shortcut strategy seems to require a small insight, the recognition that, on this type of arithmetic problem, answers can be obtained without executing the arithmetic operations.

Starkey and Gelman (1982) reported that children as young as 3 years of age can solve simple inversion problems, in particular $1 + 1 - 1$ and $1 - 1 + 1$. As the investigators noted, however, the data (percentage of correct answers) did not allow determination of whether children were adding and subtracting all three numbers or whether they were using the shortcut.

Bisanz and LeFevre (1990) devised a chronometric method for distinguishing between these strategies. They presented two types of arithmetic problems: *inversion problems* ($a + b - b$) and *standard problems* ($a + b - c$). They also systematically varied the size of b on both types of problems. The logic was that if children used the shortcut

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strategy, the size of b would not affect solution times on inversion problems (because b would not be added and subtracted). In contrast, if they added and subtracted, solution times would increase with the size of b , just as they would on standard problems on which adding b was necessary to solve the problem.

Group-level analyses indicated that all age groups tested (first graders to adults) tended to solve inversion problems faster than standard ones. However, analyses of individual performance revealed that, between first and fourth grade, only a minority of children (approximately 40%) relied on the shortcut approach. Some of the other children added and subtracted all numbers (*the computation strategy*). Others used a third approach that Bisanz and LeFevre (1990) labeled the *negation strategy*. Negation involved adding $a + b$, typically by counting up on one's fingers but then putting down all of the fingers simultaneously and saying a . As would be expected from the nature of this procedure, the solution times that it generated on $a + b - b$ problems were faster when b was small than when it was large, but the difference in times was less than when children used the computation strategy (because they only added b rather than adding and subtracting it).

Use of the shortcut strategy requires not only knowing the inversion principle but also choosing to use it on the particular problem. This recognition seems likely to be influenced by the probability of encountering such problems. Presumably, the more consistently and frequently inversion problems are presented, the stronger the activation of the shortcut strategy and the more often it will be chosen over computational approaches. Consistent with this hypothesis, German 8- to 10-year-olds who were presented 100% inversion problems were more likely to be classified as using the shortcut strategy on inversion problems than were peers who were presented 50% inversion problems (Stern, 1992, 1993).

All of these prior studies of inversion problems were based on analyses in which a given child was classified as using or not using a given strategy over the entire set of problems. When strategy use on other arithmetic tasks has been assessed on a trial-by-trial basis, however, individual children have been found to use a variety of approaches (e.g., Geary, 1990, 1995; Siegler, 1987a, 1987b). Thus, children's strategy use on inversion problems seemed likely to be more variable than these chronometric studies indicated.

Inversion problems have a property that made them especially promising both for examining strategy use on a trial-by-trial basis and for studying the role of consciousness in strategy discovery. The strategy that the problem solver uses on each trial can be assessed through two separate measures. One of the measures, immediately retrospective verbal reports, has been used in many previous studies of arithmetic. Results of these studies have demonstrated that children generally provide valid reports of their arithmetic strategies when asked, immediately after they state the answer, "How did you solve that problem?" (Geary, 1995; Siegler, 1987a, 1987b).

The unusual feature of inversion problems was that the explicit, conscious measure of strategy use provided by

verbal reports could be supplemented by an implicit measure of strategy use on each trial: the solution time. Ordinarily, solution times are insufficient for inferring the strategy that generated the time on a given trial. However, for assessing young children's strategies on inversion problems, solution times are considerably more revealing. The reason is that times on these problems are distributed bimodally; they are very fast when the shortcut is used and much slower when addition and subtraction are used. As described in the Method section, this allowed us to obtain independent measures of implicit and explicit use of the shortcut.

Conscious Insight and Strategy Discovery

This ability to independently assess strategy use on each trial through solution times and through verbal reports, together with the microgenetic design, allowed us to investigate an issue of particular importance: the relation between conscious insight and strategy discovery. In many accounts, both traditional and contemporary, discovery of new problem-solving techniques has been viewed as implying a prior conscious insight (e.g., Buehler, 1908; Gick & Lockhart, 1995). In other accounts, strategy discovery has been viewed as first arising at an implicit, unconscious, procedural level and only later at an explicit, conscious, metaprocedural one (Karmiloff-Smith, 1992; Kuhn et al., 1995). Within these latter accounts, conscious insight should follow, rather than precede, discovery of new strategies. Lack of separate measures for providing comparable trial-by-trial assessment of unconscious implicit strategy use and conscious explicit strategy use has hampered empirical investigation of this issue.

Some indirectly relevant evidence, however, is available in studies of gesture-speech concordance. These studies indicate that gestures sometimes reveal knowledge more advanced than that shown in speech (Alibali & Goldin-Meadow, 1993; Church & Goldin-Meadow, 1986; Perry, Church, & Goldin-Meadow, 1988). Children who display the advanced gestures are more likely to learn from instruction than are children who do not. This may simply mean that gestures are a particularly sensitive measure of early cognitive competence. However, it also may have the more general implication that strategies emerge at an unconscious, nonverbal level before they emerge at a conscious, verbal level. Obtaining both solution time and verbal report data on each trial promised to help discriminate between these interpretations by allowing comparison of when the shortcut strategy was first used according to the verbal report measure and when it was first used according to a nonverbal index other than gesture (the solution times).

The separate measures of implicit and explicit strategy use, together with the microgenetic design, also allowed us to address a second enduring question about discovery: whether discoveries are made abruptly or gradually. This issue of abruptness is related to—but distinct from—the issue of consciousness. Typically, theorists who emphasize unconscious processes view discovery as a gradual process, whereas those who emphasize conscious processes tend to view it as abrupt. However, these are not the only relations

that are possible, nor are they the only ones that have been posited. Some theorists have hypothesized that unconscious processes are involved in discovery but that the discoveries are nonetheless abrupt. For example, Simonton (1995, p. 477) proposed that "when the succession of subconscious images chances upon a bona fide insight, core consciousness will suddenly change focus and spotlight the discovery."

Some theories, both venerable and modern, have depicted discoveries as arising from sudden flashes of insight, as in the story of Archimedes in the bathtub (Duncker, 1945; Gick & Lockhart, 1995; Kohler, 1925; Perkins, 1995). Consistent with this view, several investigators have found that people's ratings of how close they are to solving insight problems stay at relatively low levels for a prolonged period of time and then rise abruptly when the discovery is made (Davidson, 1995; Metcalfe, 1986). These data are striking. However, they are based on conscious evaluations of internal states. Gradual unconscious progress toward discoveries may not be accessible to introspection.

Other theorists view discoveries as rising gradually through the operation of processes such as spreading activation (Bowers, Regehr, Balthazard, & Parker, 1990; Isaak & Just, 1995; Ohlsson, 1992; Yaniv & Meyer, 1987). Evidence for this view comes from demonstrations of incubation effects, in which substantial delays separate the time when people encounter relevant information and when they make the related discovery. In one such demonstration, Kaplan and Simon (1990) found that when an experimenter surreptitiously provided hints about the answer to an insight problem, the hints often exercised their effects hours after they had been presented. Such effects usually have been interpreted as reflecting subconscious, random recombination of relevant information during the intervening interval (Bowers et al., 1990; Yaniv & Meyer, 1987). However, other interpretations have also been advanced, such as the original information being insufficient to produce the solution until supplemented by additional information encountered in the environment (Seifert, Meyer, Davidson, Patalano, & Yaniv, 1995).

Again, the microgenetic method and the separate measures of implicit and explicit discovery of the shortcut strategy promised to yield unique, relevant data. The two measures together would indicate whether unconscious discovery of the shortcut strategy generally preceded conscious discovery, how long the gap was when there was a gap, and what factors influenced the length of those gaps that existed.

In the current study, German second graders (8- and 9-year-olds) were presented arithmetic problems for eight sessions. Session 1 was used to identify children who did not already use the shortcut strategy. These children were randomly assigned to either the blocked problems group or the mixed problems group. In Sessions 2–4, children were presented the types of problems specified for their group: 100% inversion problems in the blocked problems group and 50% inversion and 50% standard problems in the mixed problems group. In Session 5, children in both groups were presented a mixed set of problems; the goal was to examine whether performance would differ as a function of the type

of problems children previously received. In Session 6, children were presented the same types of problems they had received in Sessions 2–4. In Session 7, they received a mixed problems set as in Session 5; the goal was to determine whether the additional experience with inversion problems led to greater generalization of the shortcut strategy. In Session 8, transfer problems were presented to assess both appropriate generalization and overgeneralization of the shortcut strategy. The organization of the study is outlined in Table 1.

Five Dimensions of Cognitive Change

The detailed data yielded by microgenetic studies suggest a conceptual framework for thinking about changes in children's thinking. Within this framework, distinctions are made among five dimensions of change: the path, rate, breadth, variability, and sources of change (Siegler, 1995). The *path of change* is the sequence of understandings (e.g., strategies, theories, or rules) that lead to mastery of a given concept or skill. The *rate of change* is how quickly a given change comes to be consistently used once it begins to be used on a given type of problem. The *breadth of change* concerns the degree of generalization of the new approach beyond its original context. The *variability of change* concerns individual differences among children in the change process. The *sources of change* concern the experiential and biological factors that set in motion all of the other aspects of change. This framework provided a basis for organizing predictions regarding changes on the inversion task.

Source of Change

Usually, strategies are viewed as being either present or absent; a child either has a strategy or does not. However, recent research on implicit memory and implicit learning (e.g., Anderson, 1990; Dienes & Berry, 1997; Roediger & McDermott, 1993) suggests that the activation of many cognitive entities varies continuously and that degree of activation influences the reportability, accessibility, and

Table 1
Overview of Study: Types of Problems in Each Session for Each Group

Session	Group	
	Blocked problems	Mixed problems
1	Inversion and standard problems and SCHO-LASTIC test	Inversion and standard problems and SCHO-LASTIC test
2	Inversion	Inversion and standard
3	Inversion	Inversion and standard
4	Inversion	Inversion and standard
5	Inversion and standard	Inversion and standard
6	Inversion	Inversion and standard
7	Inversion and standard	Inversion and standard
8	Inversion, standard, and generalization	Inversion, standard, and generalization

transfer of knowledge. It seemed likely that the same holds true for strategy discovery.

This belief led us to hypothesize that the strategy discovery process would vary greatly depending on whether the source of change was exposure to the blocked problems set or exposure to the mixed problems set. Presenting the blocked problems set, in which the shortcut strategy was applicable on all 20 problems, was expected to lead to higher levels of activation of the shortcut strategy than presenting the mixed problems set, in which the shortcut strategy was applicable only on half of the 20 problems. If so, the blocked problems set should lead to (a) more rapid discovery of the shortcut strategy at both nonverbalizable and verbalizable levels, (b) fewer inversion problems separating the two levels of discovery, (c) a higher percentage of use of the shortcut strategy once it is discovered, (d) a higher percentage of appropriate generalization of the shortcut strategy to novel types of problems in which the strategy is applicable (e.g., $a + b - a$), and (e) a higher percentage of inappropriate generalization of the shortcut strategy to novel types of problems in which it is inapplicable (e.g., $a + b + b$).

Path of Change

Bisanz and LeFevre (1990) and Stern (1992, 1993) demonstrated that children use three strategies on inversion problems: computation, negation, and shortcut. Consideration of the hypothesized role of activation suggested that two other strategies would also be used: the *unconscious shortcut strategy* and the *computation-shortcut strategy*. The five strategies are ordered from least advanced to most advanced in Table 2. The hypothesized unconscious shortcut strategy involves the combination of a solution time of 4 s or less, a verbal report indicating use of a computational strategy, and no overt behavior. It would emerge if children used the shortcut strategy to solve the problem but were unaware that they had done so. If this strategy appeared, it would be expected to appear and to be most prevalent shortly before children began to verbalize use of the shortcut strategy. Performance should invariably be correct on such trials, because children generated the answer by relying on the shortcut (even though they were unaware of having done so).

The second hypothesized approach was the computation-shortcut strategy. It would involve a combination of a solution time greater than 4 s and a verbal report indicating use of the shortcut strategy. It would emerge if children started to use a computational strategy but, after partially executing it, realized that they did not need to add and subtract. Answers on these problems also should always be correct (because they were generated by the shortcut), and overt behavior should be common (because children started to solve the problem via computation).

Although the relative sophistication of the five strategies is generally clear, whether the unconscious shortcut strategy or the computation-shortcut strategy should be viewed as more advanced depends on the evaluation criteria. In terms of speed and accuracy, the unconscious shortcut strategy would be more advanced. Both strategies would generate 100% accuracy, but the unconscious shortcut strategy would be faster, because it would not require computation to generate the answer. On the other hand, in terms of knowledge required to use the approaches, the computation-shortcut approach would be more advanced. Unlike the unconscious shortcut strategy, it required an explicit verbal report of use of the shortcut to solve the problem. This requirement also made it likely that among children who used both strategies, initial use of the unconscious shortcut approach would precede that of the computation-shortcut approach.

Assuming that children do, in fact, use the hypothesized unconscious shortcut strategy, we would be able to test a particularly interesting prediction regarding conscious and unconscious discoveries: Unconscious strategy discoveries should precede conscious ones. The logic was the following: If strategy discoveries reflect gradually increasing activation of the new strategy, and if lower levels of activation are needed for nonreportable uses of a new strategy than for reportable ones, then children should use the unconscious shortcut, which does not involve reporting the shortcut, before they use either the computation-shortcut or the shortcut strategy, both of which require explicit recognition of its use.

The overlapping waves perspective (Siegler, 1996), derived from past studies in which strategies were assessed on

Table 2
Examples and Definitions of Strategies

Strategy	Typical overt behavior on $18 + 5 - 5$	RT (s)	Definition of strategy	
			Explanation	Overt behavior
Computation	" $18 + 5$ —19, 20, 21, 22, 23 [putting up fingers one at a time]— $23 - 5$; 22, 21, 20, 19, 18 [putting down fingers one at a time]; it's 18"	>4	Computation	Present or absent
Negation	" $18 + 5$ —19, 20, 21, 22, 23 [putting up fingers one at a time]— $23 - 5$; it's 18"	>4	Negation	Present or absent
Unconscious shortcut	None	≤4	Computation or negation	Absent
Computation-shortcut	" $18 + 5$ —19, 20, 21—oh, it's 18"	>4	Shortcut	Present or absent
Shortcut	None	≤4	Shortcut	Absent

Note. RT = response time.

a trial-by-trial basis, suggested several other predictions regarding the path of change. Children were expected to use multiple strategies on inversion problems within a given session. They also were expected to move from greater reliance on less advanced strategies to greater reliance on more advanced strategies. In particular, they were expected to rely predominantly on the computation strategy initially, to increase their use of the negation strategy relatively early in the study, to increase their use of the unconscious and computation-shortcut strategies somewhat later, and to increase their use of the shortcut strategy yet later.

Rate of Change

Bisanz and LeFevre (1990) and Stern (1992, 1993) reported a surprisingly small amount of shortcut strategy use, along with a relatively slow increase in that use, during the elementary school years. For example, in Stern's (1992) mixed problems condition, only 13% of second graders and 45% of fourth graders were assessed as using the shortcut strategy. These findings were based on assessments of each child's predominant strategy. It seems likely that many children used the shortcut strategy sometimes but not often enough for it to be apparent in their overall pattern of solution times. This reasoning, together with observations from previous microgenetic studies, suggested that most second graders in the present study would sometimes use the shortcut strategy from early in the study and that its use would gradually increase beyond that level. As noted earlier, the increase was expected to be more gradual in the mixed condition and less gradual in the blocked condition.

Breadth of Change

Stern (1992, 1993) found that the shortcut strategy was used more consistently to solve inversion problems when the problems were presented under blocked conditions than when they were presented under mixed conditions. However, her problem set did not include items that were superficially similar to inversion problems but on which the shortcut strategy was not applicable (e.g., $a + b + b$). The continuous activation hypothesis suggested that blocked presentation of inversion problems would lead to both broader appropriate generalization of the shortcut approach and more overgeneralization of it than would the mixed problems condition.

Variability of Change

Bisanz and LeFevre (1990) and Stern (1992, 1993) assessed individual differences by trying to identify the predominant strategy used by each child. The present approach allowed a more nuanced examination of individual differences. For example, it allowed us to compare changes over sessions in the number of strategies used by individual children in the blocked and the mixed problems conditions. Also, to the extent that children in the blocked problems condition increasingly relied on the shortcut strategy in the later sessions, they would be expected to use fewer strategies

in the later sessions than children in the mixed problems condition.

Method

Participants

The original sample included 39 second graders who attended an afterschool center in Munich, Germany. Among them, 8 used the shortcut strategy on at least one pretest trial and therefore did not participate further in the study. The remaining 31 children (18 girls and 13 boys; mean age = 8.75 years, range = 100 to 115 months at the beginning of the study) were randomly assigned to the two groups. Three female research assistants, ranging in age from 22 to 26 years, conducted the testing in all eight sessions.

Session 1 (Pretest)

The pretest included two parts. In the first, children were presented 20 addition problems, half of the form $a + b - b$ (inversion problems) and half of the form $a + b - c$ (standard problems). The first number in each problem ranged from 8 to 50, and the second and third numbers ranged from 3 to 18. Addition of the first two numbers in each problem always involved a carrying operation; the purpose was for the problems to yield substantially longer solution times when children computed than when they used the shortcut. Each problem was written horizontally in the form " $N + N - N = ?$ " on a 10×20 -cm index card.

In the second part of the pretest, which was administered on another day, children were presented the SCHOLASTIC test of mathematical performance (Stern, 1997). This test had previously been normed on a representative sample of more than 1,000 second graders in Munich. It includes one section on arithmetic word problems and another that tests the speed with which children can solve arithmetic computation problems.

Mean performance on the SCHOLASTIC test of children who participated in the subsequent sessions was close to previously established means: 44th percentile on word problems and 47th percentile on computation problems. Results on the test were used to assign children to the two groups. Within each quartile of test scores, equal numbers of children were assigned randomly to the blocked problems and mixed problems conditions.

Sessions 2-7 (Practice)

Children in the mixed problems group received 10 inversion problems and 10 standard problems in each practice session. Children in the blocked problems group received 20 inversion problems and no standard problems in Sessions 2, 3, 4, and 6; they received the same problems as the children in the mixed problems condition in Sessions 5 and 7. In each session, a different set of problems was used; the problems were always ordered randomly.

Session 8 (Transfer)

In the transfer session, children in both groups were presented 48 problems, 8 each of the following six types: $a + b - b$, $a - b + b$, $a + b - a$, $a - b - b$, $a + b + b$, and $a - b + a$. Appropriate generalization of the shortcut strategy would allow children to use the strategy on the $a - b + b$ and $a + b - a$ problems, which they had not seen, as well as on the $a + b - b$ problems, which they had. The simplest inappropriate generalization would lead to their ignoring the second and third numbers and answering a on all six kinds of problems rather than only on the two types of problems for

which a was correct. Other forms of inappropriate generalization were also possible, such as always saying the number that did not match the two identical numbers. These forms, however, rarely occurred.

Problem sizes were approximately equal in all eight sessions, with the mean sum of the three numbers in each problem ranging from 53 to 59. Problem sizes were also approximately equal for inversion and standard problems, with the respective means being 56 and 58. The sums of individual problems ranged from 23 to 86 on the inversion problems and from 15 to 93 on the standard problems.

Procedure

All children were tested once per week in a quiet room in their afterschool center. At the beginning of Session 1, they were told that they would be presented 20 problems on index cards. They then were told that the most important goal was to answer correctly but that they should answer as quickly as possible too. They also were told that they would be asked, after each problem, how they figured out the answer. In subsequent sessions, the instructions were repeated in abbreviated form.

Children could not see what was written on each index card until the experimenter turned it over. When the experimenter did so, she also started a stopwatch; when the child answered, she stopped it. Because no theoretically important issues depended on differences of tenths or hundredths of seconds, times were recorded to the nearest second.

After the child answered, the experimenter asked "How did you figure out the result?" If a child's explanation was ambiguous or vague (e.g., "I saw it" or "I figured it out"), the experimenter asked for more details. For example, if a child said "I figured it out," the experimenter asked "How did you figure it out?" The experimenter wrote down, immediately after each trial, the child's answer, solution time, and immediately retrospective explanation of strategy use, as well as descriptions of any visible or audible behavior that occurred during the trial. Audiotapes provided backup data on all measures except visible but inaudible overt behavior.

Children advanced three types of explanations: computation, negation, and shortcut. Table 3 provides typical examples of each of the three, all of the examples coming from children's explanations on a single problem, $35 + 8 - 8$. Computation explanations indicated that the child added the first two numbers and subtracted the third number from the sum. Negation explanations indicated that the child added the first two numbers but answered without explicitly subtracting the third number from the sum of the first two. Shortcut explanations indicated that the child realized that the

answer would equal the first number without adding the second number or subtracting the third.

If, during a session, a child used the shortcut strategy on three consecutive inversion problems, as measured by rapid solution times and explanations that indicated use of the shortcut, the experimenter did not ask for further explanations during the session. Children who met this criterion were credited with using the shortcut strategy on subsequent trials within the session on which their times were 4 s or less. The logic was that repeatedly asking for such explanations might confuse children and make them think they were proceeding incorrectly. Even when children met the three-consecutive-trials criterion, requests for explanations were reinstated at the beginning of the next session.

Definitions of Strategy Use

Strategy use was defined in terms of solution times, explanations, and, for some strategies, absence of overt computation. The solution times showed a distinctly bimodal pattern. On 39% of trials, times were 1–4 s; on 53% of trials, times were 8 s or higher. On only 8% of trials were times 5, 6, or 7 s. This led us to make a qualitative distinction between trials with times of 4 s or less, on which we inferred that the shortcut strategy was used to solve the problem, and trials with times longer than 4 s, on which we inferred that one of the computational strategies was used. Consistent with this assumption, overt computation was seen on 0% of trials when solution times were 4 s or less (as would be expected if children used the shortcut on these trials), versus 66% of trials on which times were greater than 4 s.¹

This bimodality of solution times was very helpful in diagnosing use of the five strategies. The computation strategy was defined by a solution time greater than 4 s and a computation explanation (Table 3). The negation strategy was defined by a solution time greater than 4 s and a negation explanation. The unconscious shortcut strategy was defined by a solution time less than or equal to 4 s, an absence of overt behavior, and a computation or negation explanation (unlike most computation and negation explanations, though, these did not refer to specific sums, because the child had not actually added and subtracted). The computation–shortcut strategy was defined by a solution time greater than 4 s and an explanation that indicated that the answer was generated by the shortcut (the explanation also could refer to the child beginning to compute earlier in the trial). Finally, the shortcut strategy was defined by a solution time less than or equal to 4 s, absence of overt computational behavior, and a shortcut explanation.

These definitions and examples of strategy use make clear the relations among the strategies. Each strategy in Table 2 is an advance over the one listed above it. Negation is an advance over computation, in that children substitute a simple subtraction ($8 - 8 = 0$) for a complex one ($43 - 8 = 35$). The unconscious shortcut is an advance over negation, in that answers are generated without any addition or subtraction. The computation–shortcut is an advance over the unconscious shortcut, in that it entails explicit recognition of having used the shortcut strategy (although it is also a regression from the unconscious shortcut in that the child does not use the shortcut until having started to add and subtract). The

Table 3
Examples of Each Category of Explanations

Explanation	Examples
Computation	"I added 8 to 35, which is 43, and then I subtracted 8 from 43, which is 35" "I computed $35 + 5 + 3$, which is 43, and then I subtracted 3, which is 40, and subtracted 5, which is 35"
Negation	" $35 + 8$ is 43, but because $8 - 8$ is zero, the result is 35" " $35 + 8$ is 43, then one sees that the result is 35"
Shortcut	"I saw that the result was 35, because $8 - 8 = 0$ " "I did not compute, I saw the result"

¹Overt behavior provided another possible measure of strategy use. However, unlike solution times and verbal reports, such behavior is not available on every trial. When overt behavior is not seen, any of the strategies might be being used. Thus, the solution time measure was a better index of strategy use, because it was available on every trial; overt behavior was useful as a converging measure when it was present.

shortcut strategy is an advance over all previous approaches, in that the child explicitly recognizes the applicability of the shortcut and uses it without adding or subtracting. Thus, the strategies were expected to appear in the order listed in Table 2.

Results

In this section, we first provide data on the validity of the strategy assessments, because the utility of microgenetic designs depends on the validity of these assessments. Next, we provide an overview of performance in the experiment. Then we consider, at a finer level of detail, findings regarding the path, rate, breadth, variability, and sources of change.

As a means of maximizing comparability of the data across groups and across sessions, all reported results involve performance on the first 10 inversion problems that the child encountered in a given session. Data are not reported for the second 10 inversion problems that children in the blocked problems condition encountered in Sessions 2, 3, 4, and 6; for the 10 standard problems that they encountered in Sessions 1, 5, and 7; or for the 10 standard problems that children in the mixed problems condition encountered in all sessions.

Validity of Strategy Classifications

Four distinct aspects of the present data attested to the validity of the strategy classifications. One type of validation came from the consistency of the inherent properties of each strategy with the relative lengths of solution times on trials in which children were classified as using that strategy. As shown in Table 4, times were substantially longer when children were scored as using the computation strategy, which required both addition and subtraction (16 s in each experimental condition), than when children were scored as using the negation or computation-shortcut strategy in which they performed only some of the adding and subtract-

ing (10 or 11 s). The much shorter solution times on unconscious shortcut strategy trials and shortcut strategy trials (2 or 3 s) also were consistent with the strategy classifications, but these were logically necessary given the definitions of the strategies.

A second source of validation of the strategy classifications came from overt behavior. Overt behavior that unambiguously indicated which strategy children used was present on the majority of trials for each of the three strategies that involved addition and subtraction. As shown in Table 4, such overt behavior was present on 78% and 83% of computation strategy trials in the two experimental conditions, 68% and 71% of negation strategy trials, and 48% and 77% of computation-shortcut trials.

A third source of validation came from parallels between two types of trials in which children were classified as using a given strategy: ones in which the overt behavior confirmed their use of the strategy and ones in which no overt behavior was present. If children were using the strategy that they were diagnosed as using, solution times for a given strategy should have been similar on the two kinds of trials. This proved to be the case. On two of the three strategies on which overt behavior could be present, the negation and computation-shortcut strategies, solution times were identical on trials on which overt computation was present and on trials on which it was absent (10 s for negation strategy trials and 11 s for computation-shortcut trials). On the third strategy, computation, which should and did take the longest to execute, times actually were a little longer when overt strategy use was absent than when it was present (17 s vs. 15 s). This last pair of solution times left little doubt that on trials in which children were classified as using the computation strategy, they were both adding and subtracting even when overt evidence of these activities was absent.

A fourth source of validation came from associations among the three measures of strategy use: verbal reports, solution times, and overt behavior. When children's solution times were less than or equal to 4 s, they explained their performance in terms of the shortcut strategy on 60% of trials; in contrast, they advanced such explanations on only 10% of trials on which their times were greater than 4 s. Overt computation was present on 74% of trials on which children said they used the computation or negation strategy and took more than 4 s to solve the problem. In contrast, such behavior was present on only 7% of trials on which children either said that they used the shortcut strategy or solved the problem within 4 s. These data, as well as the other three types of evidence, indicate that the strategy assessments were valid indexes of strategy use.

Overview of Data

Table 4 presents data on each strategy's frequency of use, speed, accuracy, and probability of being accompanied by overt behavior. The data are presented separately for the two experimental groups. Three facts stand out: (a) Children in both groups used all five hypothesized strategies; (b) frequency of use of the strategies differed between the groups; and (c) each strategy's accuracy, speed, and likelihood of

Table 4
Performance of Children in Mixed Problems and Blocked Problems Groups

Strategy	% use	% correct	% overt	Mean response time (s)
Mixed problems group				
Computation	40	74	78	16
Negation	27	100	68	10
Unconscious shortcut	9	100	0	3
Computation-shortcut	9	100	48	11
Shortcut	16	100	0	3
Blocked problems group				
Computation	38	73	83	16
Negation	7	100	71	10
Unconscious shortcut	17	100	0	3
Computation-shortcut	3	100	77	10
Shortcut	35	100	0	2

being accompanied by overt behavior were highly similar for the two groups. Thus, the experimental manipulation influenced how often strategies were used but not whether they were used at all or how they were executed.

Consider the specific findings regarding frequency of use of the strategies. Children in the blocked problems condition tended more often to use the more advanced strategies, and children in the mixed problems condition tended more often to use the less advanced strategies. In particular, children in the blocked condition used the most advanced strategy, the shortcut strategy, more often than did children in the mixed condition, $t(29) = 4.42, p < .001$. They also used the unconscious shortcut strategy more often, $t(29) = 3.97, p < .001$. Children in the mixed problems condition used the second least advanced strategy, the negation strategy, more often than did children in the blocked problems condition, $t(29) = 10.68, p < .001$. They also used the computation-shortcut strategy more often, $t(29) = 5.09, p < .001$. No difference between groups was present in use of the least advanced approach, the computation strategy.

In addition to more often using the more advanced strategies, children in the blocked problems condition solved problems more quickly (mean solution time = 8 s vs. 10 s), $t(29) = 3.90, p < .001$. In large part, the difference was due to children in the blocked condition more often using the two fastest strategies, the shortcut and unconscious shortcut approaches (52% of trials vs. 25% of trials for children in the mixed problems condition). No differences in accuracy were present between children in the two groups. Those in both groups answered correctly on 90% of trials.

We next consider at a finer grain of analysis the path, rate, breadth, and variability of change. Findings regarding sources of change are distributed among the four sections, because whether children encountered mixed or blocked problems affected all of the other aspects of change.

Path of Change

Changes over sessions in frequency of use of strategies. One way of looking at the path of change is to examine shifts in the frequency of use of different strategies on the inversion problems. Figure 1 presents such frequencies for children in the two experimental conditions. The light to dark dimension within the bar graphs corresponds to the sophistication of the strategies, with the lightest shading representing the least advanced strategy (computation) and the darkest representing the most advanced strategy (shortcut). Thus, the darker the bar as a whole, the more advanced the strategies that children used. This representational format shows, at a glance, the most pronounced trends in the data, as well as allowing careful analysis of the details.

First consider the general trends that are apparent in Figure 1. In Session 1, in which children in both groups were presented the mixed problems set, the two least advanced approaches, computation and negation, were the only ones used, and their frequency of use was similar in the two groups. During Sessions 2, 3, and 4, children in both groups moved toward greater use of the more advanced strategies. However, the trend proceeded more rapidly and reached a

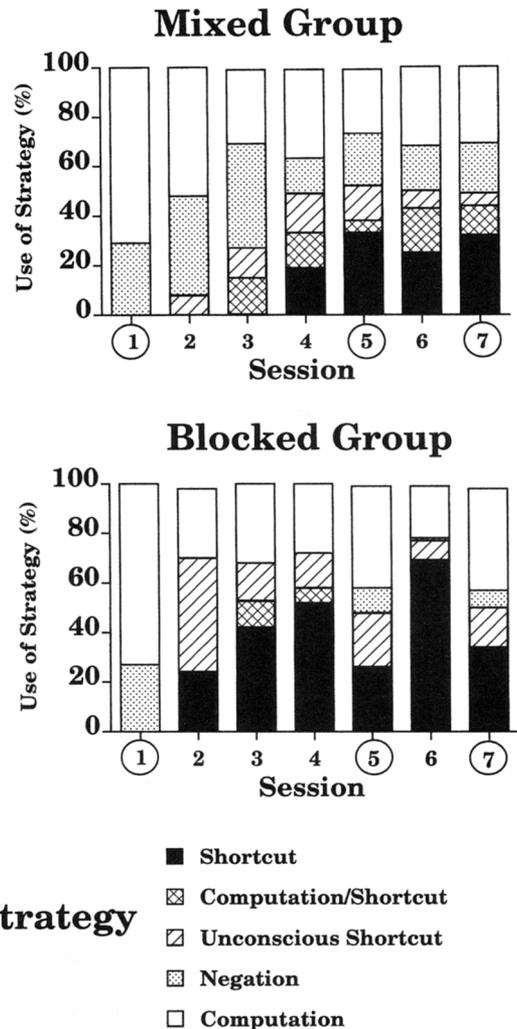


Figure 1. Changes in strategy use over seven sessions. The darker the shading, the more advanced the strategy. Thus, the increasingly dark shading in later sessions indicates increased use of more advanced strategies. Circles around Sessions 1, 5, and 7 indicate that children in the blocked problems and mixed problems conditions received identical problems in those sessions.

higher level in the blocked problems condition, in which children received 100% inversion problems in these sessions, than in the mixed problems condition, in which children received 50% inversion and 50% standard problems. In Session 5, children in both experimental conditions received 50% inversion and 50% standard problems. Children in each group used more advanced strategies on the inversion problems in these sessions than they had when presented the same distribution of problems in Session 1, but children in the blocked problems condition used less advanced strategies than in Session 4. In Session 6, when children in the blocked problems group again received 100% inversion problems, their reliance on the most advanced strategies exceeded all previous levels. On the other hand, children in the mixed problems group, who continued to

receive 50% inversion problems as they had in all previous sessions, performed much as they had in the immediately preceding sessions. Finally, in Session 7, children in both groups received 50% inversion problems, as they had in Session 5. As in that prior session, the two groups generated similar patterns of strategy use, with children in the blocked problems condition relying on less advanced strategies than they had in Session 6.

The changes over sessions in percentage use of the most advanced approach, the shortcut strategy, are especially revealing about the path of change (Figure 2). Children in the blocked problems condition used the shortcut strategy on a far higher percentage of inversion problems than did children in the mixed problems condition in Sessions 2–4, $t(28) > 3.93$, $ps < .001$. These were sessions in which they received 100% inversion problems and children in the mixed problems condition received 50% inversion and 50% standard problems. In Session 5, when children in both groups received a mixed problems set, their use of the shortcut strategy did not differ. In Session 6, when children in the blocked problems condition again received 100% inversion problems, they used the shortcut strategy even more often than they had in Session 4, and the difference between their use of it and that of children in the mixed problems group was larger than ever, $t(28) = 5.53$, $p < .001$. In Session 7, when children in both groups again received a mixed problems set, their frequencies of use of the shortcut strategy were again indistinguishable. Thus, children in the blocked problems condition learned to use the shortcut strategy quite consistently when only inversion problems were presented, but they did not use it nearly as often when standard problems were presented on half of the trials.

Order of discovery of strategies. Another facet of the path of change was the order in which different strategies were used for the first time. Figure 3 provides such data. In the blocked problems condition, more than 80% of children first used the computation strategy, then the negation

strategy, then the unconscious shortcut strategy, and, finally, the shortcut strategy. Half of these children later used the computation–shortcut strategy; half did not.

Children in the mixed problems group started similarly, with 80% first using the computation strategy, then the negation strategy, and then the unconscious shortcut approach. However, two thirds of children in the mixed problems group used the computation–shortcut approach before the shortcut strategy. Thus, most children in both groups discovered the strategies in an order that closely paralleled their hypothesized order of sophistication. The only exception was that almost all children in the blocked problems group discovered the shortcut strategy before using the computation–shortcut approach. This exception was probably due to the computation–shortcut approach rarely being used by children in this group; they used it on only 3% of trials, and half of the children never used it.²

Conscious and unconscious discovery of the shortcut strategy. Because we could assess strategy use on each trial both via a child's verbal explanation and via the child's solution time, we could determine whether children used the shortcut strategy to generate their answer before they explicitly recognized that they had used it. In particular, we could determine whether children began to show the characteristic very short solution times before their explanations reflected this use.

The results were striking. Of the 16 children in the blocked condition, 14 used the unconscious shortcut before either the computation–shortcut or shortcut strategy (the two strategies that included shortcut explanations). Of the remaining 2 children, 1 used the unconscious shortcut but never used the other two approaches, and 1 used the shortcut strategy before the unconscious shortcut. Thus, for 15 of the 16 children in the blocked problems group, the short solution times characteristic of the shortcut approach preceded the child's verbal acknowledgment of having used the shortcut. This was not due to shortcut explanations requiring some verbal facility that children lacked; 15 of the 16 children generated shortcut explanations on other trials during the study. Rather, the data indicated that, for children in the blocked problems condition, unconscious use of the shortcut strategy preceded conscious recognition of its use.

The pattern was similar in the mixed problems condition. Of the 15 children in that group, 12 used the unconscious shortcut before either of the two strategies that involved

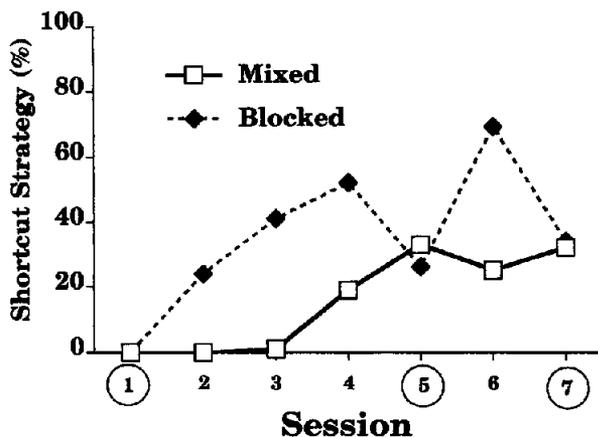


Figure 2. Percentage use of shortcut strategy in each session among children in the mixed problems condition and the blocked problems condition. Circles around Sessions 1, 5, and 7 indicate that children in the blocked problems and mixed problems conditions received identical problems in those sessions.

²In addition to the 27 children whose paths are illustrated in Figure 3, 1 child used the shortcut strategy before the unconscious shortcut, and 3 children never used the shortcut strategy. In Sessions 2, 3, 4, and 6, children in the blocked problems condition received 10 inversion problems that were not included in the data analyses reported in this article (Trials 11–20 of those sessions). We checked whether any children first used the unconscious shortcut strategy or the shortcut strategy during those second 10 trials within a session. We found that none of them did. All children in the blocked problems group first used the unconscious shortcut strategy in the first 10 problems of Session 2. All children in the group who used the shortcut strategy initially used it in the first 10 trials of Session 2 (12 children) or in the first 10 problems of Session 3 (2 children).

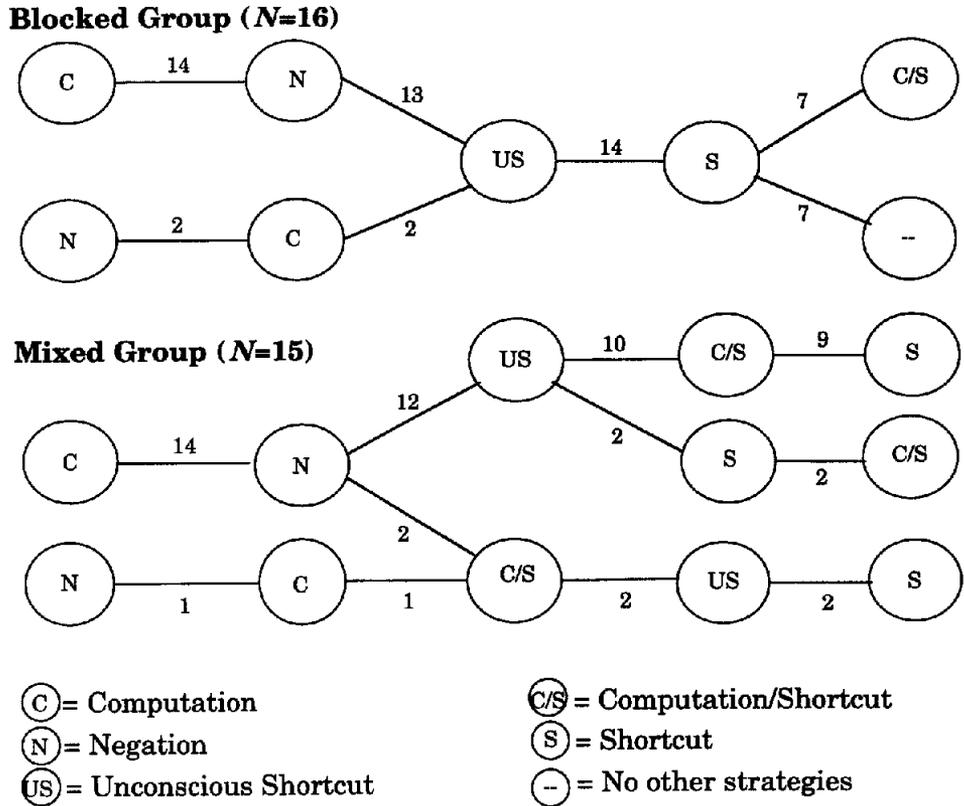


Figure 3. Sequence of first use of strategies. The first strategy used appears on the left; the last strategy used appears on the right. Letters within circles indicate the strategy that was used; numbers between circles indicate the number of children in the group who used the two strategies in that order. Thus, the 14 between the C and the N at the top left indicates that 14 children in the blocked problems group first used the computation strategy and then used the negation strategy. For purposes of simplicity, only the five main sequences of strategy discovery, used by 27 of the 31 children, are shown. This is why the numbers do not always sum to the total number of children in the group.

shortcut explanations. Of the other 3 children, 2 used the computation–shortcut strategy before using the unconscious shortcut, and 1 used only the computation–shortcut. Thus, combining the blocked and mixed conditions, 27 of the 31 children (88%) generated the short solution times that defined implicit use of the shortcut strategy before they generated verbal explanations explicitly reporting use of the strategy.

Figure 4 illustrates the length of the gap (measured in sessions) between when a child first generated a solution time of 4 s or less and when that child first generated a verbal explanation recognizing use of the shortcut (for the 27 children who generated short solution times before shortcut explanations). The gaps were considerably shorter in the blocked problems condition than in the mixed problems condition. As shown in Figure 4, 86% of children in the blocked problems condition discovered the unconscious and conscious forms of the shortcut strategy in the same session, versus 8% of children in the mixed problems condition. For the 86% of children in the blocked problems condition who first used the unconscious and conscious strategies in the same session, an average of 4.24 trials separated the two discoveries. The shorter gap in the blocked problems

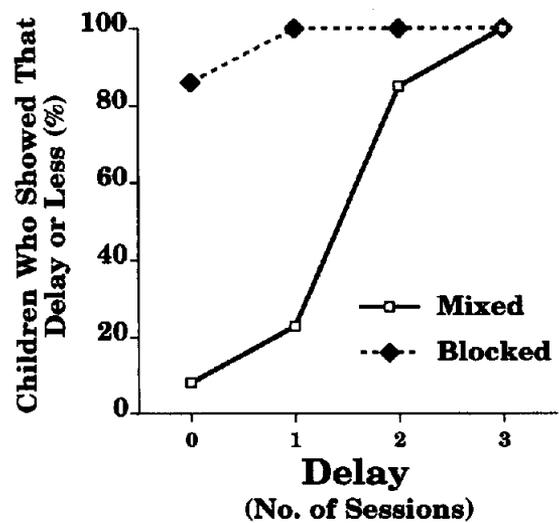


Figure 4. Percentage of children with different numbers of sessions between their first use of the unconscious shortcut strategy and their first use of the shortcut strategy. A delay of zero sessions indicates that the first use of both strategies occurred in the same session.

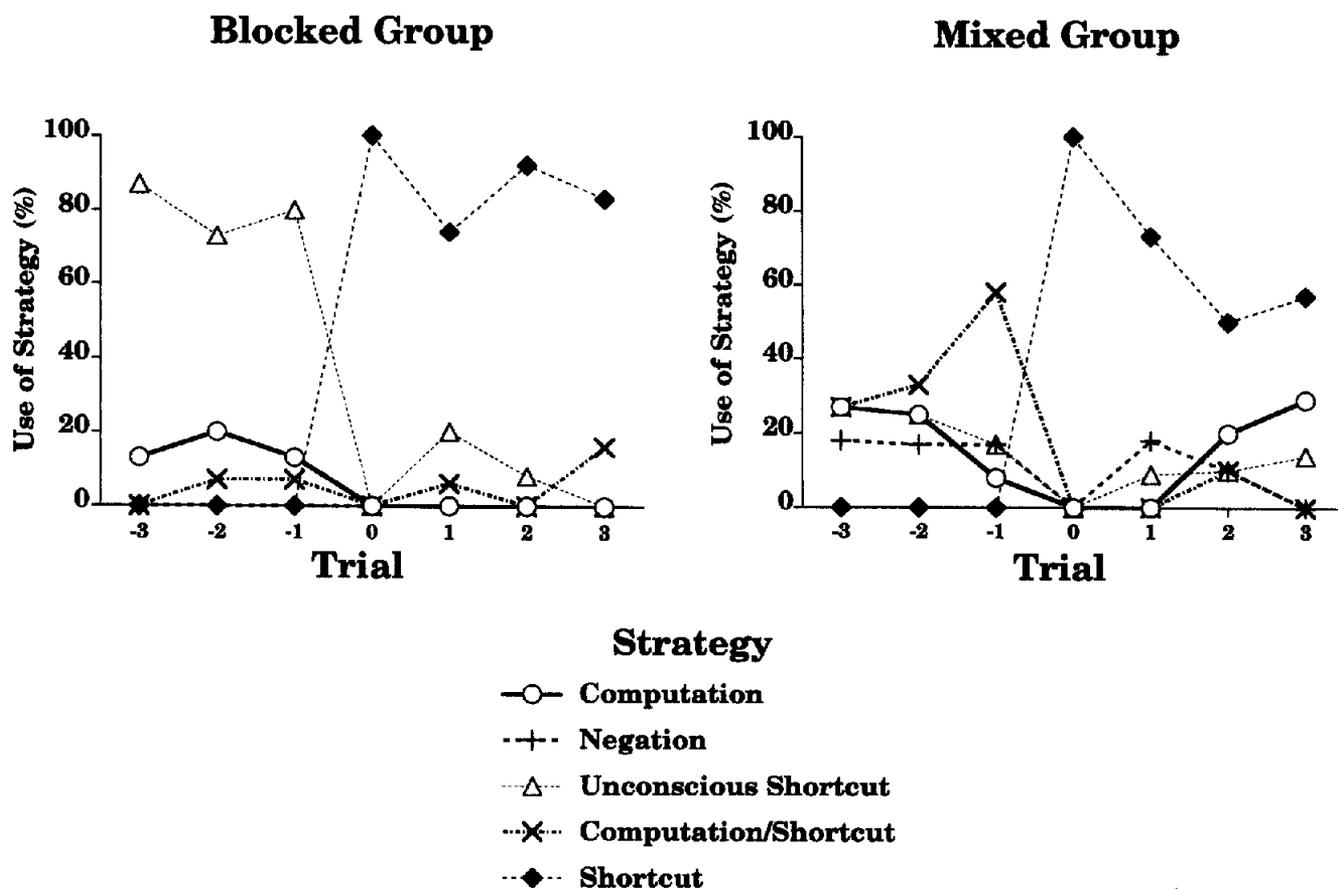


Figure 5. Strategy use on trials immediately before and after first use of the shortcut strategy. Each child's first use of the shortcut strategy was set to Trial 0. Thus, by definition, on Trial 0, 100% of children used the shortcut strategy. Performance on the -3 trial indicates the percentage of children who used each strategy three trials before their first use of the shortcut strategy, performance on the -2 trial indicates strategy use two trials before the discovery, and so on. For example, in the blocked problems condition (left panel), 87% of children used the unconscious shortcut strategy three trials before their first use of the shortcut strategy and 13% used the computation strategy. All data are from trials within the session in which the shortcut strategy was first used.

condition was consistent with the view that the more consistent presentation of inversion problems in the blocked problems condition led to greater activation of the shortcut strategy and that this increased activation led to children more quickly realizing that they were using the shortcut.

Strategy use just before and just after discovery of the shortcut strategy. The trial-by-trial assessment of each child's strategy use allowed us to assess the path of change immediately preceding and following discovery of the shortcut strategy. Figure 5 shows this micro-level path of change for the 15 children in the blocked problems condition and the 13 children in the mixed problems condition who discovered the shortcut strategy. The trial on which a child first used the shortcut strategy was that child's Trial 0. Trials -3, -2, and -1 show strategy use on the three trials just before each child's discovery, and Trials 1-3 show strategy use on the three trials just after the discovery.³

As shown in Figure 5 (left), roughly 80% of children in the blocked problems condition used the unconscious shortcut strategy on each of the three trials preceding their

discovery of the shortcut strategy. The very rapid solution times on these trials showed that the children used the shortcut to solve the problem, but their verbal reports indicated that they were unaware of having done so. After children in this condition first used the shortcut strategy, about 80% of them continued to use it on the next three trials.

³All data in Figure 5 reflect performance within the session in which the shortcut strategy was first used. Thus, if a child first used the shortcut strategy on the 8th inversion problem in a session, the child's strategy use would be included for Trials 1 and 2 (the 9th and 10th inversion problems) but not for Trial 3 (because that would be in the next session). The reason for limiting the window to the 3 trials before and after the discovery was that discoveries of the shortcut strategy tended to occur late in the session. All discoveries of it occurred on the 5th inversion problem of the session or later. With only 10 inversion problem trials within each session, too few children would have been included on trials further after the discovery for the percentages to have been meaningful.

The pattern leading up to discovery of the shortcut strategy was quite different in the mixed problems condition (Figure 5, right). The unconscious shortcut strategy was used on only 23% of problems in the three trials before discovery of the shortcut strategy. This was more than the 9% of trials on which children in the mixed condition used it on all trials but far less than the 80% use in the blocked problems condition. Conversely, the computation–shortcut strategy was used just before the first use of the shortcut far more often in the mixed problems condition than in the blocked problems one; 54% of children in the mixed problems condition used it on the trial just before the discovery (although, as shown in Figure 3, most of them had used the unconscious shortcut strategy before the computation–shortcut approach). Like the unconscious shortcut strategy, the computation–shortcut made sense as a transitional approach, because it involved children starting to solve a problem via addition and subtraction, realizing during the solution process that the computation was unnecessary, and citing the inversion principle as the basis of their solution.

On the trials immediately after discovery of the shortcut strategy, use of the approach declined more rapidly among children in the mixed problems group than it did among

children in the blocked problems condition. It was used by 73% of children in the mixed problems condition on the trial immediately after they first used it but by only about 50% of children on the next two trials.

As shown in Figure 6, children’s strategies were very different when they returned a week later for the next session (the session after the one in which they first used the shortcut strategy). First consider performance of the children in the blocked problems condition (Figure 6, left). On the first two problems of the new session, the large majority of these children used the computation strategy. Then the unconscious shortcut became the most common. Finally, the shortcut strategy became the most common. Thus, even though these children, by definition, had used the shortcut strategy in the previous session, they apparently needed to rediscover it.

When children in the mixed problems condition returned for the session after the one in which they had first used the shortcut strategy, they required even more trials than children in the blocked problems condition before most resumed using that approach (Figure 6, right). On none of the first six trials of that session did even 25% of children use the shortcut strategy. By the end of the session, however, almost

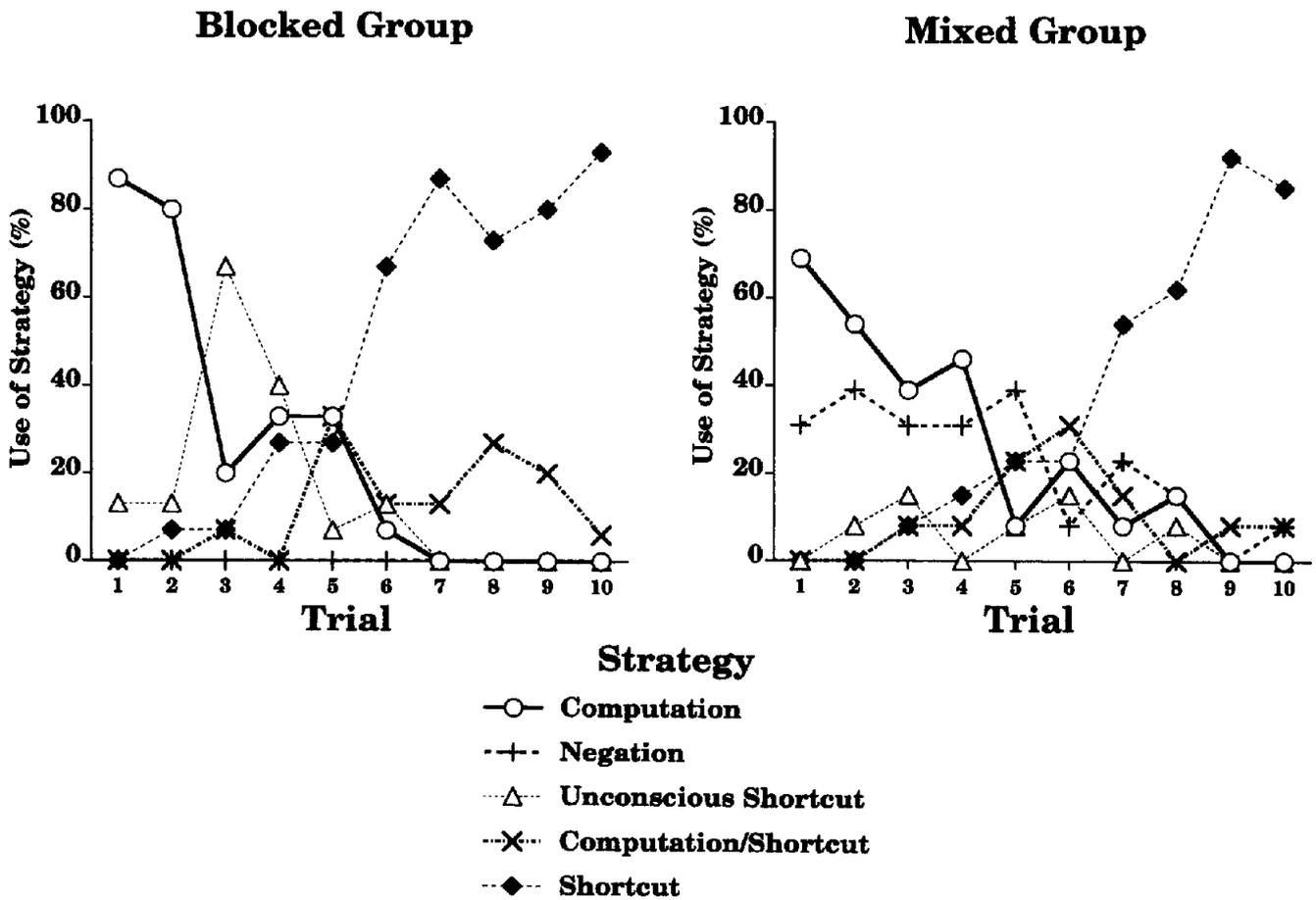


Figure 6. Strategy use in the session after the one in which children first used the shortcut strategy.

all of the children in the mixed problems condition were again using it.

Strategy use immediately preceding and following discovery of the unconscious shortcut strategy. Just as the trial-by-trial assessment of each child's strategy use allowed us to analyze the path of change just before and just after discovery of the shortcut strategy, it also allowed us to analyze discovery of the unconscious shortcut strategy. Again, children's paths of discovery in the two conditions differed substantially. In the blocked problems condition (Figure 7, left), all children used the computation strategy on the three trials preceding the first use of the unconscious shortcut strategy. After they used the unconscious shortcut for the first time, the large majority of children used it on each of the next three trials. By the next trial, 50% used the unconscious shortcut and 50% used the shortcut strategy. Overall, 80% of children in the blocked problems condition first reported that they used the shortcut strategy once or

more within five trials of when their solution times indicated that they first used it.

In contrast, the first use of the unconscious shortcut strategy by children in the mixed problems group was an isolated event (Figure 7, right). Both before and after that first use, the large majority of children used the computation or the negation strategy. Neither the unconscious shortcut nor the shortcut strategy was used often on the trials immediately after the first use of the unconscious shortcut.

Rate of Change

Even after children began to use the shortcut strategy, they increased their use of it only gradually. As expected, the rate of change was faster in the blocked problems condition, in which the strategy was applicable on all trials, than in the mixed problems condition, in which it was applicable on

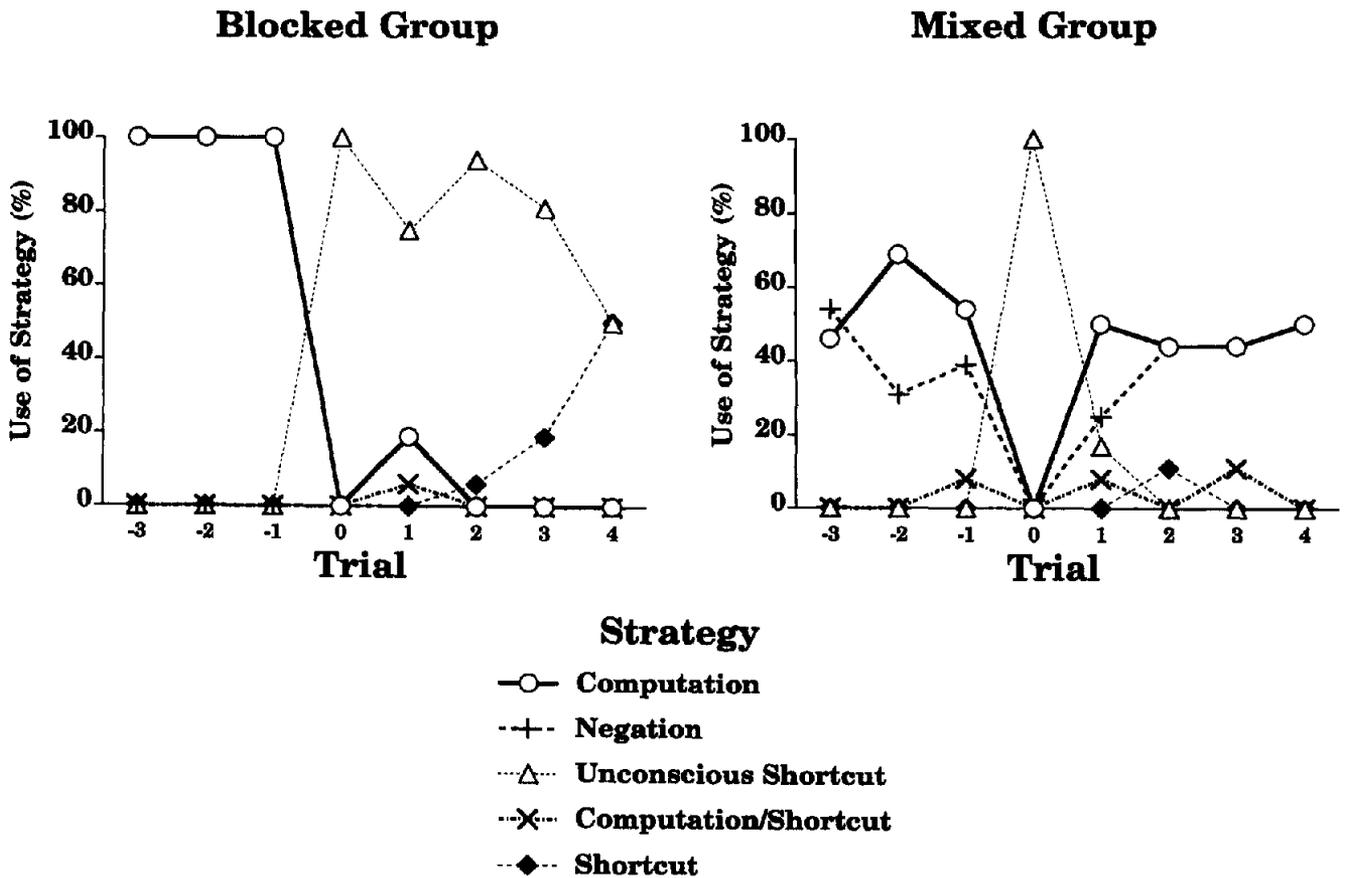


Figure 7. Strategy use on trials immediately before and after first use of the unconscious shortcut strategy. Each child's first use of the unconscious shortcut strategy was set to Trial 0. Thus, by definition, on Trial 0, 100% of children used the unconscious shortcut strategy. Performance on the -3 trial indicates the percentage of children who used each strategy three trials before their first use of the unconscious shortcut, performance on the -2 trial indicates strategy use two trials before the discovery, and so on. For example, in the blocked problems condition, 100% of children used the computation strategy three trials before their first use of the unconscious shortcut strategy. All data are from trials within the session in which the unconscious shortcut was first used.

only half. Even in the blocked problems condition, though, the rate of increase in use of the shortcut strategy was gradual.

In the blocked problems condition, 81% of children began to use the shortcut strategy during Session 2 and an additional 13% during Session 3. As shown in Figure 8, however, most children in the group did not use the shortcut strategy on a majority of trials until Session 6. In no session did more than 20% of children use it on all 10 inversion problems.

Children in the blocked problems condition showed more rapid changes in use of some of the other strategies. From Session 1 to Session 2, their use of the computation strategy declined from 73% to 28% of trials, and their use of the negation strategy declined from 27% to 0% of trials (Figure 1). The unconscious shortcut strategy served as a brief-lived transitional approach; its use increased rapidly from Session 1 (0%) to Session 2 (46%) before declining to 15% in

Session 3 and staying at similar levels thereafter. With these exceptions, though, changes in strategy use were quite gradual.

In the mixed problems condition, changes in use of the shortcut strategy came even more gradually. Children in this group took a long time to begin using the shortcut strategy. A substantial majority (69%) first used it in Session 4, and most of the others (23%) first used it in Session 5. Thus, they consistently discovered the strategy later than children in the blocked problems condition. In addition, children in the mixed problems condition never did use the shortcut strategy consistently (Figure 8). In Sessions 5, 6, and 7, almost all children in the group used the shortcut strategy on some trials, but only 1 child (7%) used it on the majority of inversion problems in any session. As shown in Figure 1, changes in use of all of the other strategies by children in the mixed problems group were also gradual.

A particularly surprising aspect of the gradual change was the continued use of the computation strategy after children discovered the shortcut strategy. This was not so hard to understand in the mixed problems condition, in which children needed to use the computation strategy to solve the standard problems. It was much more surprising, however, in the blocked problems condition. Even though the shortcut strategy was applicable to all problems in this condition in Sessions 2, 3, 4, and 6, and despite the shortcut strategy solving problems more accurately and much more quickly than the computation strategy, children continued to use the computation strategy quite often throughout the study (Figure 1).

Considering the data from Session 4 may convey the counterintuitive nature of this result. By then, 15 of 16 children in the blocked problems condition had used the shortcut strategy, and children had encountered 100% inversion problems in the preceding two sessions. Yet, children in the blocked condition still used the computation strategy on 30% of trials in Session 4. This was not attributable to a few children using the computation strategy consistently and most others not using it at all; 13 of the 16 children in the blocked problems condition used the computation strategy during the session. Thus, children continued to use the slower and less accurate computation strategy (with which they started the study) well after they had used—and explicitly reported using—the faster and more accurate shortcut strategy.

An equally striking aspect of the gradualness of the changes in strategy use involved children's explanations. Even after children's explanations explicitly acknowledged use of the shortcut strategy, children used the approach without this recognition on other trials. That is, they continued to use the unconscious shortcut strategy even after they were also using the shortcut strategy. The pattern held for children in both experimental conditions, although those in the blocked problems condition showed it more strongly. In the sessions after the one in which they used the shortcut strategy for the first time, children in the blocked condition used the unconscious shortcut strategy on 16% of trials; children in the mixed problems condition used it on 7% of trials. Of the 28 children who ever used the shortcut strategy, 26 used the unconscious shortcut strategy at least once in a

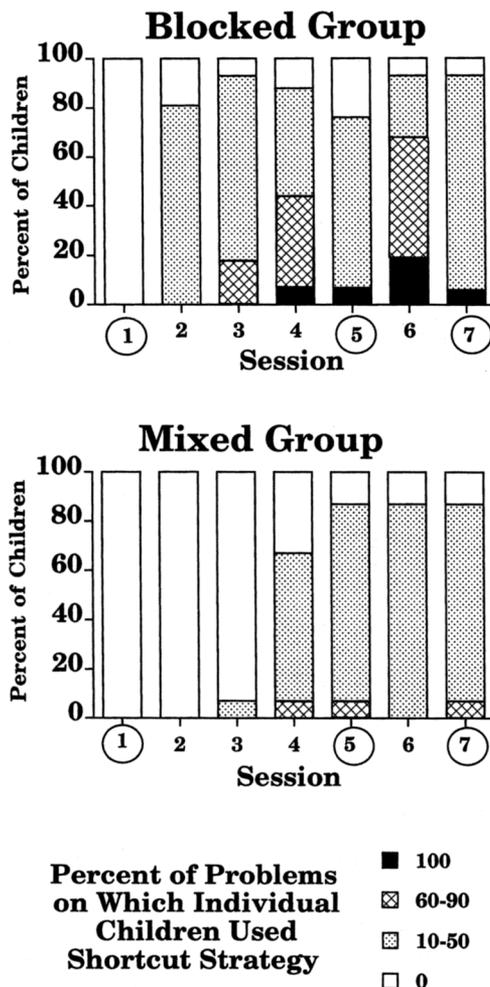


Figure 8. Percentage of children using the shortcut strategy on a given percentage of problems in a given session. Thus, the dotted area in Session 2 for the blocked problems condition indicates that 80% of children used the shortcut strategy on between 10% and 50% of trials in that session.

session after the one in which they first reported using the shortcut approach. Thus, use of the shortcut increased gradually, and explicit recognition of its use increased even more gradually.

Breadth of Change

Presenting a wide range of problems in Session 8, including ones that superficially resembled the inversion problems but did not allow the same solution, provided a test of the breadth of change. In particular, it allowed us to examine both the degree to which children appropriately generalized the shortcut to unfamiliar types of problems on which it was applicable ($a - b + b$ and $b + a - b$) and the degree to which they inappropriately generalized it to unfamiliar types of problems on which it was inapplicable ($a + b + b$, $a - b - b$, and $b - a + b$). Appropriate generalization was defined by a correct answer and a solution time of 4 s or less on unfamiliar types of problems on which the shortcut strategy was applicable. Inappropriate generalization was inferred when a child advanced the specific incorrect answer that would have been produced by applying the shortcut strategy to a problem on which it was inapplicable (e.g., $38 + 5 + 5 = 38$).

The ideal pattern of performance would have been to use the shortcut strategy consistently when it was applicable and to use it rarely or never when it was inapplicable. As shown in Table 5, this ideal pattern of transfer was rare in both groups. Only 13% of children in the mixed problems group and 25% in the blocked problems group used the shortcut strategy on two thirds or more of trials on which it was applicable and on one third or less of trials on which it was not. Even with a looser criterion of appropriate generalization—use of the shortcut strategy on more than one third of trials on which it was applicable and less than one third of trials on which it was not—only 33% of children in the mixed problems condition and 31% in the blocked problems condition generalized appropriately.

The reasons for the relatively low rates of appropriate generalization differed for the two groups. As hypothesized,

prior exposure to the blocked problems led to frequent appropriate generalization but also to frequent inappropriate generalization. Prior exposure to the mixed problems led to less inappropriate generalization, but it also led to less appropriate generalization. Specifically, children in the blocked problems condition used the shortcut strategy on a higher percentage of generalization trials on which it was applicable (65% vs. 38%), $t(28) = 2.61, p < .01$. However, they also used it on a higher percentage of trials on which it was inapplicable (66% vs. 28%), $t(28) = 3.07, p < .01$. Differences between the two groups were significant on each of the individual generalization problems except the $a - b + b$ problems, and even there, children in the blocked problems condition used the shortcut strategy considerably more often (74% vs. 53%). Thus, although the blocked problems condition produced greater use of the shortcut strategy and more frequent generalization of it to novel problems on which the strategy was applicable, it produced greater inappropriate transfer of the strategy as well.

Variability of Change

Although almost all children used multiple strategies in every session, they varied in how many they used. To examine determinants of within-child variability in strategy use, two Group \times Session analyses of variance (ANOVAs) were run on the number of strategies used by each child in each session. One ANOVA included the sessions in which children in the two groups received identical problems (Sessions 1, 5, and 7); the other included the sessions in which they received different problems (Sessions 2, 3, 4, and 6).

The 2×3 ANOVA for the sessions with identical problems indicated a main effect for session, $F(2, 58) = 55.21, p < .001$, and an interaction between group and session, $F(2, 58) = 8.59, p < .001$. Scheffé tests indicated that, in both groups, number of strategies was higher in both Sessions 5 and 7 than in Session 1. As shown in Figure 9, the number of strategies used by each child increased more in the mixed problems condition. The number of strategies used by children in the two groups did not differ in Sessions 1 and 5 but did differ in Session 7, $t(29) = 3.29, p < .001$.

The 2×4 ANOVA on number of strategies used in sessions in which children in the two groups received different problems (Sessions 2, 3, 4, and 6) showed main effects for session, $F(3, 87) = 7.38, p < .001$, and group, $F(1, 29) = 4.26, p < .05$, and an interaction between the two variables, $F(3, 87) = 18.42, p < .001$. The number of strategies increased over sessions, children in the mixed problems group used a greater number of strategies per session, and the difference between groups emerged relatively late in the study. As shown in Figure 9, the number of strategies used by children in the two groups was quite similar in Sessions 2 and 3, but children in the mixed problems condition used a greater number of strategies in Sessions 4 and 6, $t_s(29) = 2.37, p < .05$, and $8.55, p < .001$.

Table 5
Transfer Session Performance: Percentage of Children Who Generalized the Shortcut Strategy Appropriately and Who Overgeneralized It

Group/correct generalization ^a (%)	Overgeneralization ^b		
	0-33	34-66	67-100
Mixed problems			
0-33	20	7	13
34-66	20	20	0
67-100	13	7	0
Blocked problems			
0-33	6	0	6
34-66	6	0	19
67-100	25	0	38

^aSaying a on $a - b + b$ and $b + a - b$ problems within 4 s. ^bSaying a on $a + b + b, a - b - b$, and $b - a + b$ problems.

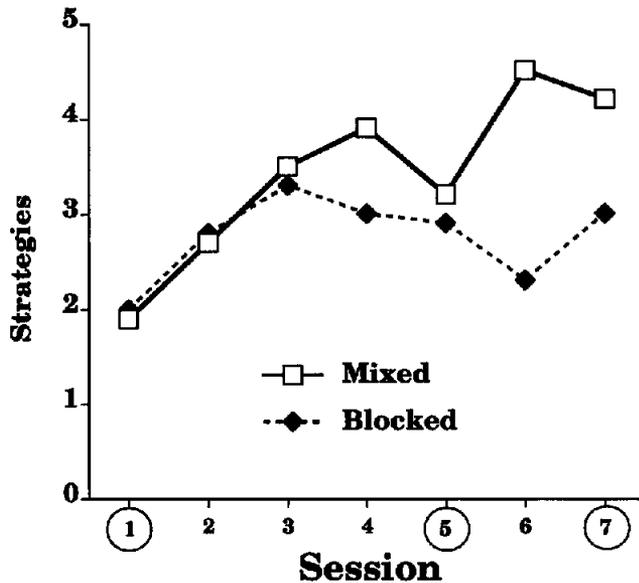


Figure 9. Mean number of strategies used in each session by children in the mixed and blocked problems conditions.

Discussion

This study focused on changes with experience in children's strategies for solving an arithmetic insight problem. At a general level, the results were what common sense—and previous studies of the inversion problem—would suggest: Use of the more advanced strategies increased over sessions; the increase was greater in the blocked problems condition; and the blocked condition produced greater transfer, both appropriate and inappropriate, of the shortcut strategy.

However, the microgenetic design and the trial-by-trial strategy assessments that were used in this study revealed that the change process was much richer and more interesting than such general descriptions could convey. In this concluding section, we first discuss the value of examining change at both coarser and finer levels of temporal resolution. Then we consider the phenomenon of unconscious strategy discovery, the role of competing activations in the strategy discovery process, and the persistence of immature strategies in the face of superior alternatives.

Value of Analyzing Change at Multiple Levels

In this study, we examined changes in children's thinking at three levels of aggregation: (a) session-by-session changes, (b) order of first use of the five strategies, and (c) trial-by-trial analyses of strategy use immediately before and immediately after discovery of the shortcut and unconscious shortcut strategies. At each level of analysis, we examined change patterns for the experimental group as a whole and for individual children.

This multilevel analysis of change served a function analogous to varying the level of magnification while looking at a cell through a microscope. Lower levels of

resolution provide a better sense of the cell as a whole; higher levels yield more precise information about its components. In the present context, the relatively coarse session-by-session analyses yielded a depiction of change over the experiment as a whole. The medium-grain order-of-first-use data indicated the number of children showing specific sequences of discoveries. The fine-grain trial-by-trial analyses yielded very precise information regarding strategy use just before and just after discoveries.

The advantages of examining change at both coarser and finer grained levels can be understood by considering the paths of change in the two experimental conditions. The session-by-session data revealed a number of similarities between the two conditions. Children in both groups started out using the computation and negation strategies. Over sessions, they increasingly relied on the more advanced strategies: the unconscious shortcut, computation-shortcut, and shortcut strategies. Children in both groups also used all five of the hypothesized strategies. And contrary to the usual depiction of cognitive change, in which children progress from Approach 1 to Approach 2 to Approach 3, almost every child in both conditions used multiple strategies in every session. New strategies were discovered, but older ones persisted as well.

The session-by-session data also revealed several differences in the paths of change of children in the two groups. Children in the mixed problems condition used the negation strategy considerably more often, especially in the early sessions, and later used the computation-shortcut strategy more often. Children in the blocked problems condition used the unconscious shortcut strategy more often, especially in the early sessions, and they increased their use of the shortcut strategy to a greater extent.

Turning up the resolution of the analysis enriched this account in several ways. The order-of-first-use data indicated that, despite the differences between groups in the amount of use of the strategies, the order in which children generated them was similar. The large majority of children in both groups first used the computation strategy, then the negation strategy, and finally the unconscious shortcut strategy.

Turning up the magnification yet higher, to scrutinize performance immediately before and after discovery of the new strategies, revealed additional similarities and differences between the paths of change under the two experimental conditions. Consider the first use of the unconscious shortcut strategy. Although the large majority of children in both groups generated the unconscious shortcut before the computation-shortcut or shortcut strategy, the significance of this event differed for children in the two groups. For children in the blocked problems condition, the first use of the unconscious shortcut marked an important transition. About 80% of children used it on each of the three trials after its initial use. Then most children began to use the shortcut strategy. In contrast, for children in the mixed problems condition, the first use of the unconscious shortcut strategy was an isolated event. They rarely used it or the shortcut strategy on the immediately ensuing trials. Instead, they

returned to heavy reliance on the less advanced computation and negation strategies.

A general lesson can be drawn from these findings: Whether a given strategy is transitional depends heavily on the conditions of learning. Developmental psychologists frequently refer to some approaches as "transitional strategies," as if they would be transitional under all circumstances. However, the present trial-by-trial data showed that a strategy that was transitional under one set of circumstances was not under another. The unconscious shortcut strategy was clearly transitional to the shortcut strategy for children in the blocked problems condition but not for children in the mixed problems condition.

Previous findings are consistent with this view. For example, several models of number conservation posit that counting the number of objects in each row is transitional between relying on the lengths of the rows and relying on the type of transformation (Piaget, 1952; Siegler, 1981; Simon & Klahr, 1995). This appears to be the case under some circumstances. Under other circumstances, however, children begin to rely on transformations without ever relying on counting (Siegler, 1995). Thus, a strategy that is transitional in some learning contexts may not be in others.

Conscious and Unconscious Strategy Discoveries

A unique feature of this study was that strategy use on every trial could be assessed at both implicit and explicit levels. The solution time on each trial provided an implicit measure of strategy use; the immediately retrospective verbal report on the trial provided an explicit measure of strategy use. The combination allowed us to examine a long-standing but difficult-to-investigate issue: whether strategies can be discovered unconsciously before they are discovered consciously.

Ericsson and Simon's (1993) thorough review of the literature on protocol analysis revealed no evidence of unconscious strategy discovery. They concluded, "We have marshalled the evidence for a simple hypothesis that is at the core of our model of human information processing: the information that is heeded during performance of a task is the information that is reportable; and the information that is reported is information that is heeded" (Ericsson & Simon, 1993, p. 167).

The present data, however, strongly indicated that, at least in the present context, nonreportable strategy discoveries occurred often. The large majority of children in both groups generated the fast solution times indicative of use of the shortcut before they explicitly reported using it. The trial-by-trial data from the blocked problems condition provided especially convincing evidence for unconscious discoveries preceding conscious ones. For most children in the blocked problems group, fewer than five trials separated their first use of the two strategies. The session-by-session data provided converging evidence. In Session 2, the session in which most children in the blocked problems condition first reported using the shortcut strategy, children in that condition used the unconscious shortcut on 46% of trials, versus 9% in the other sessions.

These data have important implications for understanding of the relation between consciousness and discovery. They clearly indicate that strategy discoveries can be unconscious. The first use of the unconscious shortcut strategy was a true discovery, in the sense that it represented a qualitative change in the solution procedure. Children in the two conditions averaged more than 12 s per trial while using the computation and negation strategies on each of the three trials before their first use of the unconscious shortcut. In contrast, on their first use of the unconscious shortcut, they averaged less than 3 s. Thus, although the discovery was unconscious, it represented a qualitative change in the solution procedure.

These data dovetail with previous evidence that children's gestures often reflect thoughts more advanced than the strategies they use to solve problems (Alibali & Goldin-Meadow, 1993; Church & Goldin-Meadow, 1986; Graham & Perry, 1993; Perry et al., 1988). In both cases, the data indicate that implicit, nonverbal indicators of knowledge emerge before verbal, explicit ones. They also are consistent with the theoretical claims of Kuhn (e.g., Kuhn et al., 1988, 1995) and Karmiloff-Smith (1992) that implicit, procedural understanding often precedes explicit, metaprocedural understanding. However, the present data go beyond the previous findings in showing that a strategy can be used to solve a problem without people knowing that they used it.

A possible objection to this claim might be that children did not really use the unconscious shortcut strategy, that its apparent use was due to our not assessing strategy use accurately. Several sources of evidence, however, argued against this view. Overt behavior provided strong converging evidence for the strategy assessments. The consistency with which children in the blocked problems condition first used the unconscious shortcut strategy a few trials before they first used the conscious version of the shortcut also argued against the assessments of the unconscious shortcut being inaccurate. The large difference between solution times on unconscious shortcut trials and those on computation and negation trials also indicated that the existence of the unconscious shortcut strategy could not be dismissed as attributable to inaccurate strategy assessment.

These data argue that the finding was not artifactual, that unconscious discoveries preceded conscious ones. The generality of the finding remains to be tested. The present data concerned the discoveries of children. Perhaps for adults, the participants in most studies reviewed by Ericsson and Simon (1993), the relation is different. Only trial-by-trial analyses of adults discoveries can determine whether this is the case. Similarly, it is not known at present whether children or adults generate unconscious discoveries on other tasks. There is little doubt, however, that in the present context, unconscious discoveries preceded conscious ones.

A large unresolved issue concerns the mechanism through which unconscious discoveries contribute to conscious ones. The data are consistent with at least two accounts. One account emphasizes differing thresholds: Unconscious discoveries reflect a lower level of activation than conscious ones, enough activation to surpass one threshold but not the other. Versions of this hypothesis have been proposed by

Bowers et al. (1990), Isaak and Just (1995), Ohlsson (1992), and Yaniv and Meyer (1987). One result from the present study that supported this interpretation relatively directly was the short gap between the first uses of the unconscious and conscious versions of the shortcut strategy in the blocked problems condition. Activation of the shortcut would presumably rise steeply in this condition, as a result of children continuously encountering problems on which it was applicable. In contrast, activation of the shortcut would take longer to rise from the unconscious threshold to the conscious one in the mixed problems condition, because the strategy was applicable on only 50% of problems.

Another explanation also was plausible, though. The conscious version of the shortcut may have arisen through children observing their own execution of the unconscious shortcut strategy and drawing inferences about it. When children used the unconscious strategy, their explanations were necessarily less detailed than if they had used the strategy they said they used. For example, when children used the computation strategy to solve $35 + 8 - 8$, they noted that $35 + 8$ was 43. However, when they used the unconscious shortcut, they had not determined what $35 + 8$ equaled; they typically just said "I added 35 and 8" or "I added and then I subtracted." Observing such vagueness in their own verbal reports, and realizing that they could not be more precise, may have led them to realize that they had solved the problem without adding and subtracting. The rapidity with which they answered when using the unconscious shortcut also may have led them to realize that they were solving the problem differently than when they computed the answer. The two explanations are not exclusive; both differing thresholds and inferences from self-observations may be involved in the relation between unconscious and conscious discoveries.

Role of Activation in Strategy Discovery

We expected that activation of the shortcut strategy would be higher in the blocked problems condition than in the mixed problems condition. A great deal of evidence from all phases of the study supported this view. More generally, the data attested to the pervasive influence of activation on strategy discovery.

All six predictions of the activation hypothesis were confirmed. Relative to the mixed problems condition, the blocked problems condition led to (a) more rapid discovery of the shortcut strategy at both nonverbalizable and verbalizable levels, (b) fewer trials separating the two levels of discovery, (c) a higher percentage of use of the shortcut strategy once it was discovered, (d) a higher percentage of appropriate generalization of the shortcut strategy to novel types of problems in which the strategy was applicable, and (e) a higher percentage of inappropriate generalization of the shortcut strategy to novel types of problems in which it was inapplicable. The sixth prediction of the activation hypothesis was that, in both experimental conditions, the unconscious shortcut, which presumably required lower levels of activation, would be generated before the conscious version

of the strategy. Again, this prediction was correct for the large majority of children.

But how did activation exercise its effects? Particularly revealing in this regard were data on the computation–shortcut strategy. This was the strategy in which children started to add and subtract but then stopped and solved the problem via the shortcut. Use of the computation–shortcut by children in both groups revealed that the shortcut and the computation strategy could both be activated sufficiently on a single trial to exceed their threshold for execution. Its use also suggested that, during the processing of a single problem, activation of the shortcut continued to build after the computation strategy was being executed and could rise to a level that led to cessation of the computation approach after it had started to be executed. This was quite different from the winner-take-all algorithms assumed by most computer simulation models, both symbolic and connectionist (McClelland, 1995; Siegler & Shipley, 1995). The much greater use of the computation–shortcut in the mixed problems condition indicated that characteristics of surrounding problems within a session, as well as characteristics of the individual problem being solved, contributed to the level of activation of each strategy on the new problem. The fact that the computation–shortcut strategy was used primarily in Sessions 3 and 4 in the blocked problems condition, the sessions immediately after most children in that condition discovered the shortcut strategy, indicated that the competition between strategies continued within individual problems in those sessions, despite all problems in Sessions 2, 3, and 4 being solvable via the faster and more accurate shortcut approach.

Analysis of the role of the computation–shortcut strategy also was revealing regarding a small anomaly within the results. Three children, all in the mixed problems condition, generated a shortcut explanation, indicative of explicit use of the shortcut, before they generated a fast solution time, indicative of implicit use of it. But how could explicit use of a strategy precede implicit use of it?

Consideration of the likely patterns of activation of the computation and shortcut approaches suggested a way in which such cases could arise. Presumably, a lower level of activation is required to generate implicit than explicit use; this is why the large majority of children generated the unconscious shortcut before the shortcut strategy. However, even if the level of activation of a strategy is above the threshold for reportable use, it may not be used if a competing strategy has yet higher activation. The computation strategy would have been an especially potent competitor in the mixed problems condition, because it would be applicable to all problems presented in that condition and the shortcut strategy would not. This greater applicability of the computation strategy may have led to some children in the mixed problems condition not using the shortcut strategy until well after its activation exceeded the threshold of reportability.

Within this view, it makes sense that all 3 children who generated a shortcut explanation before a fast solution time first generated the explanation in the context of the computation–shortcut strategy rather than the shortcut strategy. The

computation strategy was sufficiently activated that it would start to be executed, even though the shortcut was also sufficiently activated to be used and reportable. It also makes sense that 13 of the 15 children in the mixed problems condition first generated a shortcut explanation in the context of using the computation–shortcut strategy (Figure 3). Computation apparently was a sufficiently strong competitor that it delayed some children's first use of the shortcut strategy beyond the time when the shortcut was sufficiently activated to be reportable.

The findings have several general implications for how competing activations influence the strategy discovery process. They suggest that when the level of activation of a strategy is moderately high, unconscious use of the strategy may occur; that higher levels of activation are required for conscious use of the strategy; that activation of a new strategy can build during the course of a trial, leading to the new strategy first appearing as an interruption in the execution of a previous approach; and that high levels of activation of existing approaches can delay the first use of a new strategy even after the activation of the new strategy has surpassed the threshold of reportability. As with the ideas regarding unconscious and conscious strategy discoveries, these conclusions regarding the role of activation in strategy discovery need to be tested in other contexts.

Persistence of Immature Strategies

Perhaps the most puzzling finding of the study was that children in both groups continued to use their main initial approach, the computation strategy, throughout the study. Continued use of the computation strategy by children in the blocked problems condition was especially surprising. By the end of Session 2, 81% of children in this condition had discovered the shortcut strategy. In Sessions 3 and 4, they continued to receive problems on which the shortcut was always applicable. The shortcut strategy was far faster than the computation approach and considerably more accurate. Yet children in the blocked problems condition, like those in the mixed problems condition, continued to use the computation strategy on about one third of trials throughout the seven sessions. How can this persistence be explained?

The data ruled out four otherwise plausible interpretations. One such interpretation was that children were not really using the computation strategy when they said that they were using it. However, the fact that solution times were so much longer on trials on which they said they used the computation strategy than on any others argued strongly against this view.

A second interpretation was that use of the computation strategy after discovery of the shortcut reflected the behavior of a minority of children. However, almost all children used the computation strategy in all sessions.

A third interpretation was that children occasionally reverted to using the computation strategy so that they could check whether the shortcut worked. Most children, however, used the computation approach at least 15 times after they discovered the shortcut strategy.

A fourth interpretation was that some type of experi-

menter demand effect was operating. However, this interpretation fails to indicate why children would think that the experimenter wanted them to use a slower, less accurate, and less insightful strategy. It also fails to account for their use of the shortcut strategy increasing over sessions.

So why did children use computational strategies after discovering the shortcut approach? Two types of explanations seem likely. One involves forgetting. During the week separating one session from another, children may have forgotten the shortcut. Consistent with this view, frequency of use of the shortcut strategy declined sharply from the end of one session to the beginning of the next and then rose again within each new session.

A second explanation is that the persistence of well-established strategies is a basic characteristic of human cognition that occurs even without week-long gaps between sessions. Within a single session, preschoolers continue to count from one after they discover the more efficient strategy of counting from the larger addend (Grupe, Huffman, & Bray, 1997; Siegler & Jenkins, 1989). School-aged children revert to using less mature map drawing, memory, and arithmetic strategies after having generated more mature ones (Alibali & Goldin-Meadow, 1993; Coyle & Bjorklund, 1997; Karmiloff-Smith, 1992). Adolescents and adults continue to use immature scientific experimentation strategies after they have generated more effective ones (Kuhn et al., 1995; Kuhn & Phelps, 1982; Schauble, 1990, 1996).

This persistence of immature strategies might seem to be a maladaptive feature of cognition. Viewed in terms of a single experimental task, it is. However, maintaining multiple approaches has many advantages. It enables people to adapt quickly to unexpected changes in the environment. Even in psychology studies, experimenters have been known to switch the type of problems—and thereby the strategy that will be most effective—without warning. In the longer term, strategies that lose their utility in one situation may later prove useful in unforeseen ways. For example, in learning single-digit arithmetic, some children use the strategy of decomposition quite often. They might solve $7 + 9$ by thinking " $7 + 10 = 17$; 9 is 1 less than 10; $17 - 1 = 16$." This strategy loses much of its utility as children learn to retrieve the single-digit facts. However, it becomes more useful later, when children need to perform multidigit mental arithmetic (e.g., solving $244 + 799$). A further reason to maintain older, seemingly less adaptive strategies is that ideas that seem insightful when generated do not always withstand the test of time; in retrospect, miracle cures are not always so miraculous. Thus, although maintenance of the computation strategy in the present situation may have been maladaptive, it may be part of a larger, generally adaptive tendency toward maintaining strategic variability.

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