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The Role of Situational Context in Solving Word Problems

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Word problems depicting the comparison of quantities have been shown to be difficult for elementary school children in several studies. One reason for this may be that young children lack situational understanding because they are not familiar with the quantitative comparison of sets. To test this assumption, we presented 45 first graders with "compare" problems that were embedded in a familiar situational context. The results showed that children who received compare problems following stories that induced situational understanding of qualitative comparisons performed better than children who received compare problems following stories that had nothing to do with the comparison of sets. The data suggest that most first graders have access to the mathematical problem-solving knowledge necessary to understand and solve compare problems only when these problem are embedded in a familiar situational context.

In several studies, it has been shown that simple arithmetic word problems that demand addition or subtraction of two numbers are not necessarily equal in difficulty (Briars & Larkin, 1984; Carpenter & Moser, 1984; Cummins, Kintsch, Reusser, & Weimer, 1988; Kintsch & Greeno, 1985; Riley & Greeno, 1988; Riley, Greeno, & Heller, 1983; Stern, in press). Problems dealing with the *comparison* of sets (e.g., John has 5 marbles. Peter has 2 marbles. How many marbles does Peter have less than John?) are more difficult than problems about the *exchange* (e.g., Peter had 3 marbles. Then John gave him 2 other marbles. How many marbles does John have now?) or the *combination* of sets (e.g., John has 5 marbles. Peter has 2 marbles. How many marbles do John and Peter have altogether?). Riley and Greeno (1988) reported that 67% of the first graders were able to solve exchange problems, 60% performed correctly in combine problems, but only 19% solved compare problems. These results indicate that

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non-mathematical factors such as language understanding and text comprehension can considerably influence the difficulty of word problems.

Research on word-problem solving has been quite extensive. Several models have been developed that simulate young children's understanding and solution of word problems that concern the exchange, combination, and comparison of sets (Briars & Larkin, 1984; Cummins et al., 1988; Kintsch & Greeno, 1985; Reusser, 1989a, 1990; Riley & Greeno, 1988; Riley et al., 1983). A common feature of all these models is that the fit between model predictions and empirical data is better for exchange and combine problems than for compare problems. Currently, no model can explain why compare problems are so difficult, nor why different kinds of compare problems differ in difficulty. Even Riley and Greeno (1988), who emphasize the role of mathematical knowledge in solving compare problems, admit that it must be text comprehension and situational understanding factors that make compare problems so hard.

In compare problems, either the difference set, the compare set, or the reference set can be looked for. For the latter two kinds of problems, the unknown set can be either the small or the large set. Altogether, there are six kinds of compare problems depicted in Table 1, with item difficulties for American first graders quoted from Riley and Greeno (1988) and German first graders quoted from

Table 1. Types of Compare Problems and Proportions of Solutions for American First Graders (from Riley & Greeno, 1988, Table 2) and German First Graders (from Stern, in press, Table 1)

	Riley	Stern
Unknown difference set		
CP1 Mary has 5 marbles. John has 8 marbles. How many marbles does John have more than Mary?	.28	.28
CP2 Mary has 6 marbles. John has 2 marbles. How many marbles does John have less than Mary?	.22	.32
Unknown compare set		
Large set unknown.		
CP3 Mary has 3 marbles. John has 4 marbles more than Mary. How many marbles does John have?	.17	.53
Small set unknown		
CP4 Mary has 5 marbles. John has 3 marbles less than Mary. How many marbles does John have?	.28	.58
Unknown reference set		
Small set unknown		
CP5 Mary has 9 marbles. She has 4 marbles more than John. How many marbles does John have?	.11	.22
Large set unknown		
CP6 Mary has 4 marbles. She has 3 marbles less than John. How many marbles does John have?	.06	.16

Stern (in press). According to Riley and Greeno's (1988) computer simulated model of word-problem solving, problems with an unknown difference set are easier to solve than the other problems because they do not require an arithmetic solution, but can be solved by establishing a one-to-one correspondence and counting the objects that remain. Nonetheless, Riley and Greeno's own empirical data show that these problems are more difficult for first graders than mathematical demands would lead them to expect.

According to Riley and Greeno (1988), understanding and solving problems with an unknown compare set and an unknown reference set requires that one establishes a mathematical equation, and therefore requires access to the *part-whole-schema* (Resnick, 1989; Resnick & Greeno, 1990; Riley & Greeno, 1988). A person is credited with representing a part-whole-schema if he or she is able to solve arithmetic problems that demand understanding the principles underlying arithmetic, such as the commutativity of addends or the complementarity of addition and subtraction (Resnick, 1989). If the difference between two sets is given in a compare problem, only children who understand that the difference between two sets can be interpreted as a set itself are able to build a mathematical equation for compare problems with an unknown compare set or an unknown reference set. The mathematical models for both kinds of problems are the same: Either the small set or the difference set are known and have to be added in order to find the large set, or the large set and the difference set are known and have to be subtracted in order to find the small set. Nonetheless, problems with an unknown reference set are more difficult than problems with an unknown compare set (Cummins et al., 1988; De Corte, Verschaffel, & De Winn, 1985; Riley & Greeno, 1988; Stern, 1991). This result again suggests that factors other than mathematics make compare problems with an unknown reference set difficult, as discussed in Stern (1991) and De Corte, Verschaffel, and Pauwels (1991).

Several models of word-problem solving emphasize the role of language comprehension (Cummins et al., 1988; Kintsch & Greeno, 1985; Reusser, 1989a, 1990) and situational understanding as the process of "going beyond the text" (Kintsch & van Dijk, 1978). According to these models, situational understanding has the function of bridging the gap between language understanding and mathematical problem solving knowledge. To find out the adequate mathematical equation, one has to understand the situation described in the problem in order to reduce it to its gist.

Especially for word problems dealing with the comparison of sets, this process seems to be considerably difficult for young children. Two reasons may explain this phenomenon:

1. Children may have difficulties with understanding *abstract language expressions*, such as "Peter has *3 marbles less than* John" or "*How many more* marbles does John have than Peter?" There is a large body of research indicating that young children do not understand, until quite later on, quantifier terms such as "less" or "more" (Clark, 1983, for review).

2. Children may lack *situational understanding* because they are unfamiliar with the comparison of sets. According to Piaget, young children's cognitive operations are related to their actions. There are action parallels for combine and exchange problems, but not for compare problems.

Hudson (1983), De Corte and Verschaffel (1985), and Davis-Dorsey, Ross, and Morrison (1991) have shown that some changes in the use of language substantially influence the difficulty of compare problems. Hudson (1983) found that whereas 96% of the 6-year-old children could solve the following problem (a), only 25% could solve the problem (b).

(a) Here are 5 birds and here are 3 worms. Suppose the birds all race over and each tried to get a worm. How many birds won't get a worm?
(b) Here are 5 birds and here are 3 worms. How many more birds than worms are there? (p. 85)

In addition to the difference in language surface structure, however, these two problems differ in the situational knowledge they require. Under the "do not get" version, children may be reminded of familiar and emotional situations such as: "There are not enough resources (e.g., cookies); some people won't get anything and I can be one of these people." In addition, the verbal formulation "How many . . . won't get . . ." is closely related to an imagined action schema: Five birds are seeking, then three worms are crawling on the floor, then three of the five birds are pecking, while two remain without worms. In contrast, under the "how many more" version, no familiar context is activated, and children may not understand the gist of the story. That is, they may not understand why it would be useful to compare the number of birds and the number of worms, and thus they cannot successfully access a numerical procedure to solve the problem.

In the studies mentioned previously, language surface structure and situational understanding factors are confounded. Therefore, it is not clear whether some word problems are so difficult because of the unfamiliarity of the story situation or the formulation of "how many more . . ." To avoid confounding language comprehension and situational understanding, we designed a study to manipulate the situational context, but not the verbal formulations of the word problems. Thus, we address the question of whether compare problems, formulated in the abstract way depicted in Table 1, become easier when they are embedded in a concrete, enriched, and familiar story context.

Cummins et al. (1988) did not find a facilitation effect from enriched story context. For example, the compare problem CP5 (see Table 1) was not easier when it was embedded in a vignette such as "Jane and Mimi play tennis together twice a week. They both always try hard to beat each other. Both of them decide to buy new tennis rackets. So far Jane has saved 13 dollars for her racket. She has saved 5 dollars more than Mimi. How many dollars has Mimi saved?" However, the reason for this result may be that the situation described in the short story has nothing to do with the savings of Jane and Mimi, nor does it refer in any way to the comparison of sets. Thus, this kind of episodic situational context may not serve the purpose of making contact with abstract problem-solving schemes.

The assumption that young children are not as familiar with the comparison of sets as they are with the exchange or the combination of sets is only partially true. Even young children are familiar with comparisons among people. For example, they carefully compare whether their siblings or friends get more sweets than they do themselves. However, in most comparisons of these kinds, it is not necessary to calculate the exact difference set. Indeed, there may be no difference in the degree of protest if a sibling gets 1 or 10 more sweets. Thus, young children are familiar with a *qualitative comparison*, which refers to the question, "Who has more (or fewer) objects," but not with a *quantitative comparison*, which refers to the question, "How many more (or fewer) objects has Subject A than Subject B?"

Therefore, to test the role of situational context in solving compare problems, we told children a story about a competitive context dealing with qualitative comparison before presenting the arithmetic word problem. Nearly every child knows from personal experience that there are other children who possess better and more things. Two children were mentioned in the story and compared in several ways. One child was always disadvantaged with respect to the other. These same two children were mentioned in the arithmetic word problem that dealt with the comparison.

If situational context facilitates the solution of compare problems, how does it work? If the only function of situational context is to "get the ball rolling," that means that in order to make contact with the abstract knowledge that guides the mathematics of the problem, the precise relation between the qualitative comparison in the story and the quantitative comparison in the word problems should not be important. However, if the subjects are aware of the situational context during the whole process of problem solving, a facilitating effect can only be expected if the context of the story and the problem are compatible. To test this question, we designed two experimental conditions: In one, the situational context was compatible; in the other, there was a contradiction between the situational context in the story and the arithmetic word problem.

METHOD

Subjects

Forty-five first graders (21 girls, 24 boys) from Munich after-school care centers participated in the study. The mean age was 6 years, 10 months. The subjects had entered school 6 months before our study was conducted.

Procedure

All six compare problems listed in Table 1 were presented to each child with a different story for every problem. The subjects were tested individually in two sessions (5–8 days time between each session) with three problems in each session. The order of problem presentation was randomly mixed. The story and the problem were read to the child. Then, the child was asked to solve the problem and to give an oral answer. Subjects were randomly assigned to one of three groups: compatible situational context group ($n = 16$), incompatible situational context group ($n = 14$), or control group ($n = 15$). The following describes the procedure for each group.

Compatible Situational Context Group. Situational knowledge about the comparison of quantities was imparted by stories involving a comparison between two people. The relation between the two people was described in competitive terms. For example: "Peter is Laura's older brother. Because he is older, his bedroom is larger and his toys are more expensive than Laura's. Peter also gets more pocket money than Laura and he has a new bike whereas Laura has Peter's old bike. When Peter does his homework, Laura doodles a little bit. Peter has 6 crayons. Laura has 4 crayons. How many crayons less does Laura have than Peter?"

Incompatible Situational Context Group. In this group, the same stories were told to children as in the compatible situational context group, but in the word problem presented at the end of the story, the child who had a disadvantaged position in the story possessed more of the described objects. For example, the story mentioned previously ended with the problem "Laura has 6 crayons. Peter has 4 crayons. How many crayons less does Peter have than Laura?"

Control Group. Subjects in the control group were told a story of the same length as that in the other two groups, but the story had no qualitative comparison between the two people mentioned in the word problem. For example: "Bert and Lydia are in the same school class. Their teacher is Mrs. Siegler. She makes a lot of nice things with the children. Yesterday they were at the zoo. Today they are painting the animals. Bert has 6 crayons. Lydia has 4 crayons. How many crayons less does Lydia have than Bert?"

RESULTS

The mean number of problems solved for the three problem types and the groups are depicted in Table 2. An analysis of variance (ANOVA) was conducted on these data with group (control, compatible, incompatible) as between-subject variables and problem type (unknown difference set, compare set, reference set) as a repeated measure. There were significant main effects for group, $F(2, 42) = 4.38$, $p < .025$, and the problem type, $F(2, 84) = 13.68$, $p < .001$, but no significant interaction. A post hoc Tukey test indicated that children in the compatible situational context group performed better than those in the control

Table 2. Means and Standard Deviations of the Number of Solved Problems

	Condition							
	Compatible		Incompatible		Control		Total	
Problem Type	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Difference	1.33	.90	1.23	.83	.57	.75	1.06	.88
Compare	1.55	.61	1.46	.66	1.28	.72	1.44	.65
Reference	1.11	.58	.61	.65	.50	.76	.77	.70
Totals	4.00	1.49	3.30	1.60	2.35	1.59	3.28	1.67

group ($p < .05$). The incompatible situational context group did not differ from the other groups at the .05 level.

Given the differences between the groups, the next question we asked was whether different kinds of mistakes were typical for different groups. From other studies (Cummins et al., 1988; De Corte et al., 1985; Riley et al., 1983), we know that there are three main types of errors: arithmetic errors, wrong operation errors (subtraction instead of addition or vice versa), and given number errors (a number mentioned in the test is answered), among others. The percentage of subjects who made one of these mistakes at least once for each problem type is depicted in Table 3. There were only a few mistakes that could not be assigned into one of the three mistake-categories but had to be classified as "others."

Table 3. Percentage of Students Who Made a Mistake in at Least One of the Two Presented Problems of Each Type

	Compatible	Incompatible	Control	Mean
Difference set unknown				
Arithmetic	11	15	7	11
Wrong operation	27	15	29	24
Given number	17	23	43	28
Other	0	7	15	7
Compare set unknown				
Arithmetic	17	30	35	27
Wrong operation	11	7	0	6
Given number	11	7	22	13
Other	0	7	0	2
Reference set unknown				
Arithmetic	22	23	22	22
Wrong operation	44	60	50	51
Given number	11	38	21	23
Other	0	0	0	0

Fisher's Exact tests were conducted for each error type and problem type separately, but there were no significant differences at the .05 level.

DISCUSSION

The results indicate that compare problems become easier when they are embedded in a situational context dealing with a qualitative comparison. Therefore, in line with the work of Kintsch and Greeno (1985), Cummins et al. (1988), and Reusser (1989a, 1989b, 1990), our data support the claim that some word problems are difficult because the children lack appropriate situational knowledge. The results clearly indicate that it is not the use of abstract language expressions such as "How many more . . ." or ". . . 3 less than . . ." *alone* that make compare problems so difficult, because under all conditions these expressions were used in the same way, but they proved less difficult in the compatible situational context group than in the other groups.

Our results do not clarify precisely the effect of the incompatible situational context. Although the statistical analyses did not reveal significant effects, it appears that under this condition children may have special difficulties with unknown reference set problems. Stern (1991) has shown that children tend to change the gist of these problems by altering the second sentence (e.g., "He (John) has 3 marbles more than Peter" into "Peter has 3 marbles more than John."). Children may be especially prone to this mistake under the incompatible context condition because it makes the story and the problem compatible. Follow-up studies considering indicators of information processing are necessary for examining this issue further.

In several studies, it has been shown that word problems become easier when they are embedded in an enriched context (Davis-Dorsey et al., 1991; De Corte et al., 1985), for example, if a subject's own name or names of their pets are used. The familiar names may cause children to pay more attention and, moreover, it is easier to remember a familiar name than an unfamiliar one. There is some evidence for such general context effects in our study, too. Children in the control group who merely received an unrelated story context performed better than children of the same age in comparable studies where no enriched story condition was provided at all (Stern, in press, as quoted in Table 1).

However, beyond such general context effects, problem solving in the compatible and the incompatible situational context groups may have been facilitated for reasons due to the acquisition of mathematical knowledge. The stories for these groups referred to comparisons between two persons that involved qualitative comparison (who has more, who has less) but not quantitative comparison (how many more or less does one person have than the other). Resnick (1989) and Resnick and Greeno (1990) have developed a theory of the acquisition of quantitative schemata, such as the compare schema. According to this theory, children learn facts about quantitative comparisons (for example, large

set = small set + difference set) by combining knowledge about qualitative comparison of sets and numerical knowledge such as basic arithmetic. Knowledge about qualitative comparison, called *protoquantitative compare schema* by Resnick (1989), is thus the source of the quantitative compare schema. Considering these ideas, one can conclude that the mental representations of qualitative and quantitative comparison knowledge are closely connected and, therefore, the activation of situational context referring to qualitative comparisons may facilitate the accessing of the quantitative compare schema. Only with experience can the quantitative compare schema be detached from its sources and become an independent knowledge structure. In the case of arithmetic compare problems, this means that the quantitative compare schema becomes directly accessible. Thus, with development, it is no longer necessary to embed compare problems in a situational context that deals with qualitative comparison. Providing this context may, however, be necessary to support the acquisition of abstract mathematical knowledge.

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