Abstract

Probably, you know Bloch equations from NMR. In NMR, Bloch equations describe the evolution of a nuclear spin in a magnetic field upon radio frequency (rf) irradiation. By analogy, optical Bloch equations (OBEs) describe the evolution of an electronic quantum system upon coherent light irradiation. Within this project we will discuss the optical Bloch equations. After a short introduction where you derive the OBEs from the Schrödinger equation you will solve them numerically and study the time evolution of a two level system.

Task

The Hamiltonian of a two level system in an rf field is

$$\hat{H} = \hat{H}_0 + \hat{H}_I$$

$$\hat{H}_0 = \hbar\omega_e |e\rangle\langle e| + \hbar\omega_g |g\rangle\langle g| = \hbar\omega_0 |e\rangle\langle e| + 0\,|g\rangle\langle g|$$

$$\hat{H}_I = -\vec{d}\cdot\vec{E}\,\cos(\omega_L t) = -\left(d_{eg}|e\rangle\langle g| + d_{ge}|g\rangle\langle e|\right) E_0\,\cos(\omega_L t) = \left(|e\rangle\langle g| + |g\rangle\langle e|\right)\hbar\Omega_1 \cos(\omega_L t)$$

$$\Omega_1 = -\frac{d_{eg}\ E_0}{\hbar}$$

$$d_{eg} = d_{ge}* = -e\langle e|\hat{x}|g\rangle$$

where ω_0 is the resonance frequency and d_{eg} the transition dipole moment. The time evolution of the density matrix is given by

$$\frac{d}{dt}\hat{\rho}(t) = \frac{1}{i\hbar}\left[\hat{H},\hat{\rho}\right]$$

$$\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i| = \rho_{ee}|e\rangle\langle e| + \rho_{eg}|e\rangle\langle g| + \rho_{ge}|g\rangle\langle e| + \rho_{gg}|g\rangle\langle g|$$

1) Write down the differential equation for each element of the density matrix.

To simplify this equation we will change the basis and use the rotating wave approximation (RWA). The elements of the density matrix transforms as followed

$$\tilde{\rho}_{eg} = \rho_{eg} e^{i\varpi_l t}$$
$$\tilde{\rho}_{ge} = \rho_{ge} e^{-i\varpi_l t}$$
$$\tilde{\rho}_{gg} = \rho_{gg}$$
$$\tilde{\rho}_{ee} = \rho_{ee}$$

where ω_l is the radiation frequency.

2) Write the equations in the new basis.

3) Which term can be neglected according to the RWA? What is the physical meaning of each term?

By adding the elements of the transformed density matrix in a particular way we can express all equations in one, containing only vectors.

$$u = \frac{1}{2}\left(\tilde{\rho}_{ge} + \tilde{\rho}_{eg}\right)$$
$$v = \frac{1}{2i}\left(\tilde{\rho}_{ge} - \tilde{\rho}_{eg}\right)$$
$$w = \frac{1}{2}\left(\tilde{\rho}_{ee} - \tilde{\rho}_{gg}\right)$$

This leads to simple form of the optical Bloch equation:

$$\dot{\vec{r}} = \vec{\Omega} \times \vec{r}$$

$$\text{where} \quad \vec{r} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad \vec{\Omega} = \begin{pmatrix} \Omega_1 \\ 0 \\ -\delta_L \end{pmatrix}, \quad \text{and} \quad \delta_L = \omega_l - \omega_0$$

The vector $\vec{r}$ is called Bloch vector, Ω_1 the Rabi frequency and δ_L the detuning.

4) What is the meaning of u, v and w? How does the Bloch vector evolve in time with respect to Ω?

The OBEs do not include spontaneous emission which leads to relaxation of the excited state to the ground state. We just include this manually

$$\dot{u} = \delta_L v - \frac{\Gamma}{2} u$$
$$\dot{v} = -\delta_L u - \Omega_1 w - \frac{\Gamma}{2} v$$
$$\dot{w} = \Omega_1 v - \Gamma\left(w + \frac{1}{2}\right)$$

where Γ is the natural line width which corresponds to a life time $\tau = \frac{1}{\Gamma}$ of the excited state.

For the time dependent calculations it is advisable to use the scaled Bloch equations by substituting $\tau := t \cdot \Omega_1$

$$\dot{u} = \frac{\delta_L}{\Omega_1} v - \frac{\Gamma}{2\Omega_1} u$$
$$\dot{v} = -\frac{\delta_L}{\Omega_1} u - w - \frac{\Gamma}{2\Omega_1} v$$
$$\dot{w} = v - \frac{\Gamma}{\Omega_1}\left(w + \frac{1}{2}\right)$$

In the scale OBE the length of one Rabi-Oscillation period on resonance is $\tau_0 = 2\pi$.

Solve the following questions 5) -13) with Mathematica. It is available on IDES for free.

5) Solve the scaled OBE for the steady-state case $\dot{r} = 0$. Use the command `Solve.` Plot u, v and w against laser detuning for different Rabi frequencies Ω and different line width Γ (use `Plot`). How does the linewidth of the graph w depend on Γ and Ω? If you want to measure the lifetime of the excited state, would you chose a high or a low laser intensity?

For the following time-dependent calculations always start with the atom in the ground state!

6) Solve the OBE with following parameter settings: $\delta_L = 0$ and $\Gamma = 0$. Plot your result and comment the time evolution of the Bloch vector. Also plot the vector component w in a 2D plot. Around which axis does the Bloch vector rotate?
   Use the command `NDSolve` to numerically solve the differential equations. To plot a 3D vector use `ParametricPlot3D`.
7) Consider the cases $\delta_L = \Omega_1$ and $\delta_L = 10\Omega_1$. How does the frequency and the rotation axis of the Bloch vector change?
8) Probably you have heard about the adiabatic theorem in a quantum mechanics course. Can you describe it in your own words? There is also a short article on Wikipedia.
9) Now solve the OBEs with an electromagnetic field where the frequency varies in time: Assume the detuning δ_L is changed linearly from $-10\,\Omega_1$ to $10\Omega_1$ in the time interval $\Delta\tau = 2\pi \times 100$. Show the time evolution of the Bloch vector in a 3D plot.
10) Assume the detuning is changed linearly from $-10\,\Omega_1$ to $10\Omega_1$in the time interval $\Delta\tau = 2\pi \times 5$. Plot the new result.
11) Explain the difference between the last two results. Which one is the adiabatic transfer?
12) We introduce a small decay due to spontaneous emission. Assume $\delta_L = \Omega_1$ and $\Gamma = 0.1\Omega_1$. Compare with the undamped system. What do you observe? How does w evolve in time?
13) Calculate the length of the Bloch vector L(t) for the case above. Try to comment on the general qualitative behavior. What effect does the damping have on the Bloch vector?

Info

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Because I am absent during the period from the 15. April until the 4. May, I will discuss the project with each group individually in the week of the 6. May. But I advise you to start solving the tasks earlier. If you are totally lost my alternate Aaron von Conta will help you. Do not hesitate to contact him.