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Formulation of a model for pore water convection in thawing subsea permafrost

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FORMULATION OF A MODEL FOR PORE WATER CONVECTION IN THAWING SUBSEA PERMAFROST

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ABSTRACT

Ice-bearing permafrost exists in a transient state under much of the Beaufort Sea shelf. A thawed layer usually underlies the sea bed. Its rate of evolution depends critically on the rate of salt transport from the sea water through the thawed layer and into the ice below. In some areas convection of the interstitial water is an important salt transport process. The field observations in one such area at Prudhoe Bay, Alaska, are reviewed. They are used as a basis for the formulation of a theoretical model whose purpose is to describe some of the features of the convective regime. The theory is complicated by coupling of heat and salt transport processes, and by a moving phase boundary. However, in some realistic cases the theory can be decoupled and simplified. Some of the features of the convective regime are known from the field observations: Peclet number for heat $\ll 1$, Peclet number for salt $\gg 1$, extremely large Rayleigh number for salt, and probably a very complicated pattern of convection.

ZUSAMMENFASSUNG

Der mit Eis durchsetzte Permafrost unterhalb des grössten Teils des Beaufort-Schildes ist in einem transienten Zustand. Gewöhnlich ist unterhalb des Meeresbodens eine aufgetaute Schicht zu finden. Ihre zeitliche Entwicklung hängt in kritischer Weise vom Grade des Salztransportes ab, der vom Meerwasser durch die aufgetaute Schicht in das darunter liegende Eis erfolgt. In gewissen Regionen bildet die Konvektion des Porenwassers einen wichtigen Faktor für den Salztransport. Es werden die Feldbeobachtungen in einem solchen Gebiet bei Prudhoe Bay, Alaska, beschrieben. Sie dienen als Grundlage zur Formulierung eines theoretischen Modelles, dessen Ziel es ist, einige der Hauptursachen dieser Konvention zu beschreiben. Die Theorie ist deshalb kompliziert, weil die Wärme- und Salztransportprozesse gekoppelt sind, und weil sich die Phasengrenzfläche zwischen dem gefrorenen und aufgetauten Permafrost bewegt. In einigen realistischen Fällen können diese Prozesse in der Theorie entkoppelt und stark vereinfacht werden. Einige der Haupteigenschaften der Konvektionsströmung sind aus den Feldbeobachtungen bekannt, indem die Pecletzahl für Tautemperatur wesentlich kleiner als 1 und die Pecletzahl für Salz wesentlich grösser als 1 sind, die Rayleighzahl für Salz ausserordentlich grosse Werte annimmt und die Konvektionsströmung wahrscheinlich eine sehr komplizierte Struktur aufweist.

INTRODUCTION

Ice bearing subsea permafrost is now known to be widespread on the continental shelves of the Arctic Ocean, a matter of considerable importance in connection with present attempts to develop petroleum resources there. The permafrost exists because the shelves where not always submerged. At times of different shoreline position they were emergent and exposed to cold sub-aerial temperatures long enough for substantial permafrost growth to occur. The present subsea permafrost regime is therefore a transient one, which is responding to relatively warm and salty oceanographic conditions. In the Beaufort sea of Alaska mean annual sea bed temperatures are negative, but a thawed layer of sediments varying from 0 to 100 or more meters in thickness has developd beneath the sea bed; thawing at the base of the subsea permafrost by geothermal heat must be occurring as well. To give some perspective, the thickness of the ice-bearing sediments onshore at the Prudhoe Bay oilfield is about 600 m, and the ice content is 40 % by volume (Lachenbruch and others, in press).

The existence of thawing at negative temperatures points to the importance of salt, the transport of which is often the rate controlling factor. It has been known for some time that in relatively permeable subsea sediments, gravity driven convection of the pore water in the thawed layer occurs, leading to efficient transport of salt through the thawed layer and rapid thawing (Harrison and Osterkamp, 1978 - hereafter referred to as H and O, 1978). The driving mechanism is the relatively fresh and buoyant water generated by the melting ice. In impermeable sediments this process does not occur and the thaw rates are extremely slow.

Because the requirement of phase equilibrium forces a connection between salinity and temperature at the bottom of the thawed layer, the convective regime has some unusual properties. The purpose of this report is to provide a physical description of the regime, and to formulate quantitative models of it. This is particularly attractive at our present stage of knowledge, because field observations have determined numerical values, or at least the orders of magnitude, of all the input parameters for such a model. It is felt that a better understanding of the convective regime will help us to solve some of the outstanding problems discussed later, such as the uniformity of the temperature along the bottom of the thawed layer.

THE FIELD OBSERVATIONS

We shall focus our attention on one relatively well-studied area where convection is occurring, near the Prudhoe Bay West Dock. Several investigators have carried out subsea permafrost observations there in the last decade. Some general information can be found in a review by Sellmann (1980). Information particularly useful for our purposes is given by Harrison and Osterkamp (1982) and Osterkamp and Harrison (1982). Most of the numerical values used later come from these sources. Near the West Dock, the base of the thawed layer, also called the phase boundary because it is a liquid-solid H_2O interface, has a strikingly well-defined parabolic shape from 440 m to several km from shore (Figure 1). It also has an extremely uniform temperature, implying a uniform salinity along it. Closer to shore the shape and temperature are more complicated. Although the phase boundary seems extremely sharp, it is known that some salt, and therefore some liquid phase, exist below it. Within the thawed layer the pore water salinity is about 25 % higher than that of normal sea water. It is fairly constant, except near the sea bed, but it shows two systematic features, a small general increase with depth, and a small decrease in the bottom fraction of a meter. The tem-

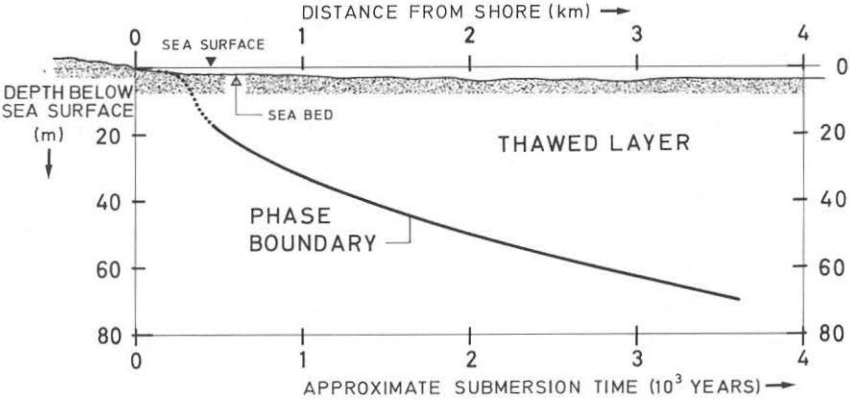

Figure 1 Approximate configuration of subsea permafrost at Prudhoe Bay, Alaska, near the West Dock. The phase boundary is shown dashed near shore where its details are not well-known. The submersion time scale is based on an assumed shore line retreat rate of 1 m yr^{-1}.

perature profiles through the thawed layer seem to be quite linear, except for the obvious effect of seasonal variations in sea bed temperature. Seasonal ice formation sometimes occurs in the shallow sea bed sediments. The present shoreline retreat rate is roughly 1 m yr^{-1}.

The reasons for some of these features are known or have plausible explanations. The parabolic shape of the phase boundary probably results from a more or less constant shore line retreat rate, so that the submersion time offshore is proportional to distance. Simple Stefan theory, which predicts that the thickness should increase as $\sqrt{time}$ under simple conditions, then predicts the shape. The more complicated behaviour near shore is not well understood, but it is partly due to a colder sea bed, which is particularly noticeable where the water depth is less than the 2 meter late winter sea ice thickness. This introduces a transient into the thawed layer development. The realtively high salinity in the thawed layer is probably due to the fact that seasonal pulses of highly saline water are generated by growth of sea ice or by salt drainage from it. This dense water can sink into the sediments, while the less saline, less dense water present at other times of the year cannot. The slight downward increase of salinity in the thawed layer may have a similar origin; the saltiest pulses may tend to sink deepest. Alternatively, the increase could be a relict effect from saltier near shore conditions. The thin layer at the bottom where the salinity decreases is a boundary layer in the conventional sense within which the salt transport changes from convective to diffusive. The linear temperature profiles suggest that the heat transport is primarily diffusive (or "conductive"); in other words, the Peclet number for heat transport $Pe_T \ll 1$. The rapid thaw rates, known from the shoreline retreat rate, indicate that the opposite is true for salt; $Pe_S \gg 1$ (H and O, 1978). These Peclet numbers can be different because the diffusivity of salt is so much less than that of heat.

Other features of the development of the thawed layer are not understood. The sea bed temperature and salinity are thought to vary considerably in both space and time, even beyond 440 m, so the uniformity of the phase boundary shape and salinity is a surprise. It is not known whether the salt below the phase boundary should be treated as an initial condition, or whether it has penetrated since submersion. Although the order of magnitude of the pore water velocity can be guessed, little is known about the structure of the velocity field. To provide the basis for prediction of that structure is one of the main purposes of the model.

GENERAL THEORY

We begin by defining all the symbols that will be used, which are unfortunately numerous.

Coordinates:

- x horizontal, perpendicular to shore
- y vertical, positive down, zero at sea bed
- z horizontal, parallel to shore
- t time
- $\hat{e}_g$ unit vector along y-axis
- Y phase boundary y-coordinate

Heat:

- Pe_T Peclet number for heat transport
- T temperature (^oC)
- κ thermal diffusivity
- γ porosity times ratio of water and bulk heat capacities
- h latent heat per unit volume of the porous medium
- K_1 thermal condictivity above phase boundary
- K_2 thermal condictivity below phase boundary
- $T_0 = T|_0$ sea bed temperature
- $T_Y = T|_Y$ phase boundary temperature
- G equilibrium geothermal gradient in permafrost onshore
- T_s ground surface temperature onshore
- T_f freezing temperature of overlying sea water with salinity S_0.

Salt:

- Pe_s Peclet number for salt transport
- S porewater salinity
- S_r a reference salinity, zero of salinity scale
- κ_s diffusivity of salt
- S' $S - S_r$
- $S_0 = S|_0$ sea bed salinity
- $S_Y = S|_Y$ phase boundary salinity
- s a characteristic salinity used in defining a Rayleigh number
- ΔS salinity change across the boundary layer at the phase boundary

Mass flow:

$\underset{\sim}{v}$	actual water velocity
$v_{x,y,z}$	velocity components
ϕ	porosity
k	hydraulic conductivity $\div\ \phi$
P	pressure, expressed as a head
ρ	density
ρ_r	a reference density corresponding to the reference salinity S_r
a	a constant expressing the dependence of ρ on S
f(T)	a function of T expressing the dependence of ρ on T
P'	$P - y$
v_c	a characteristic water velocity
v_F	the flux velocity
Ra	Rayleigh number
v	velocity along the phase boundary
b	boundary layer thickness

The theory is summarized in a fairly general way, so the subsequent approximations will be more obvious. It is as follows:

$$\nabla\cdot(\phi\underset{\sim}{v}) = 0\,, \qquad \text{Incompressibility,}$$

$$\underset{\sim}{v} = -k(\nabla P - \rho/\rho_r\,\hat{e}_g)\,, \qquad \text{Darcy's law,}$$

$$\nabla\cdot(\kappa_s\,\nabla S) - \underset{\sim}{v}\cdot\nabla S - \frac{\partial S}{\partial t} = 0\,, \qquad \text{Salt transport,}$$

$$\kappa\,\nabla^2\,T - \gamma\,\underset{\sim}{v}\cdot\nabla T - \frac{\partial T}{\partial t} = 0\,, \qquad \text{Heat transport,}$$

$$\rho/\rho_r = 1 + a\,(S - S_r) + f(T)\,, \qquad \text{Equation of state.}$$

There are two main assumptions in these equations, that the sediments are isotropic, and that no inertial terms are needed in Darcy's law. The latter needs to be checked after more is known about the velocity solution. Additional assumptions are as follows: that the porosity (ϕ) and salt diffusivity (κ_s) are constant, that heat transport is primarily conductive

($\gamma\ \underset{\sim}{v} \cdot \nabla T$ negligible), and that the effect of temperature variations on density is negligible ($f(T)$ negligible). The latter two assumptions seem to be quite valid. Further simplification results from combination of the second and fifth equations. We then have

$$\nabla \cdot \underset{\sim}{v} = 0,$$

$$\underset{\sim}{v} = -k(\nabla P' - a S' \hat{e}_g),$$

$$\kappa_S \nabla^2 S' - \underset{\sim}{v} \cdot \nabla S' - \frac{\partial S'}{\partial t} = 0,$$

$$\kappa \nabla^2 T - \frac{\partial T}{\partial t} = 0,$$

in which the unknowns are $\underset{\sim}{v}$, P', S' and T. In solving these equations the hydraulic conductivity to porosity ratio (k) would be assumed constant as well. The most important aspect of these simplified equations is that the temperature enters only into the last, and is therefore decoupled from the other variables as far as the governing equations are concerned. However, we will see that it is still coupled via the boundary conditions.

Boundary conditions apply at the sea bed and at the phase boundary. In the most general case they involve derivatives normal to these surfaces. Except close to shore it is probably a reasonable approximation to assume these surfaces are horizontal, thereby simplifying the complex geometry to a horizontal infinite layer. z-dependence of the boundary conditions is also ignored; this is also a reasonable approximation. At the phase boundary the boundary conditions then become

$$v_y\big|_Y = 0\,, \qquad \text{Impermeable boundary,}$$

$$S\big|_Y \propto -T\big|_Y\,, \qquad \text{Phase equilibrium,}$$

$$S\big|_Y \frac{dY}{dt} = -\kappa_S \frac{\partial S}{\partial y}\bigg|_Y\,, \qquad \text{Stefan condition for salt,}$$

$$h\,\frac{dY}{dt} = -K_1 \frac{\partial T(y<Y)}{\partial y}\bigg|_Y + K_2 \frac{\partial T(y>Y)}{\partial y}\bigg|_Y\,, \qquad \text{Stefan condition for heat.}$$

The third condition implies that there is no salt transport below the phase boundary. The second sometimes needs to be corrected for the effect of pressure. At the sea bed the boundary conditions are

$$v_x\Big|_0 = 0$$
$$S\Big|_0 = \text{prescribed function of } x \text{ and } t$$
$$T\Big|_0 = \text{prescribed function of } x \text{ and } t.$$

The first of these implies a permeable sea bed, where the overlying free water does not sustain any pressure gradient. It can be stated in the equivalent form $P'|_0 =$ constant. In fact, travelling waves on the surface of the ocean do cause sea bed pressure variations that induce pore water motion and contribute to salt transport. The phenomenon has been investigated theoretically by Harrison and others (1982) for the case in which gravity-driven convection is absent, and found to be negligible when the hydraulic conductivity has the magnitude characteristic of Prudhoe Bay. It therefore seems likely that gravity driven convection is the dominant process, so the constant pressure boundary condition should suffice.

Initial conditions corresponding to permafrost conditions before submersion also have to be specified. Then $Y = 0$, and the temperature has the simple form

$$T = Gy + T_S$$

determined by the equilibrium geothermal gradient (G) and the surface temperature (T_S). $T =$ constant is a sufficient approximation for some cases.

An example of how this complicated theory permits exact solution when $\underset{\sim}{v} = \underset{\sim}{0}$ and the boundary and initial conditions are simple is given by H and O (1978). This is the purely diffusive salt transport limit $Pe_S = 0$.

FORMULATION OF THE MODEL

Before we can justify the further approximations that lead to the simplest useful models, the physics of the development of the thawed layer needs to be discussed carefully. Let us assume that the phase boundary is flat, and focus our attention on one particular place, now offshore, where for simplicity the sea bed temperature (T_0) and salinity (S_0) are assumed fixed in time. As long as $T_0 > T_f$, where T_f is the freezing temperature of seawater with salinity S_0, thawing will occur if salt can penetrate. It is the nature of that penetration, whether by molecular diffusion or by motion of the pore water, and how it is influenced by the requirement of phase equilibrium at the phase boundary, that we need to discuss.

Figure 2 shows the T and S distributions across the thawed layer for different transport regimes. Curves a, calculated from H and O (1978), are for a purely diffusive regime ($Pe_S = 0$) which has the following chain of consequences:

$$\begin{matrix} \text{large} & & \text{low} & & \text{warm} & & \text{low} & & \text{low} & & \text{slow} \\ & \Rightarrow & & \Rightarrow & & \Rightarrow & & \Rightarrow & \text{heat} & \Rightarrow & \\ \nabla S & & S_Y & & T_Y & & \nabla T & & \text{flow} & & \text{thawing} \end{matrix}$$

in which the connection between S_Y and T_Y is forced by phase equilibrium. In this diffusive case the thaw rate is very slow because the temperature gradient is very small. On the other hand, if convection is occurring it will tend to smear out the large diffusive salt gradient (∇S) and alter the above chain of events. Curves b show what might be expected for medium convective development ($Pe_S \approx 1$). Curves c show the situation for strongly developed convection ($Pe_S \gg 1$). The thaw rate is largest in this case because the temperature gradient is largest. The bottom of the salinity curve c is magnified in Figure 2 to show that a thin boundary layer, in which the transport is diffusive, must exist as long a the phase boundary is impermeable. However, the salinity jump across the thin boundary layer is very small, and the salinity at the phase boundary is essentially the same as at the sea bed. This leads to an important simplification, because when the convection is not strongly developed the phase boundary temperature and salinity have to be determined as part of the solution of the full coupled problem with Stefan boundary conditions. The temperature curves in Figure 2 are drawn linear to indicate that the heat transport, even in case c, is diffusive ($Pe_T \ll 1$).

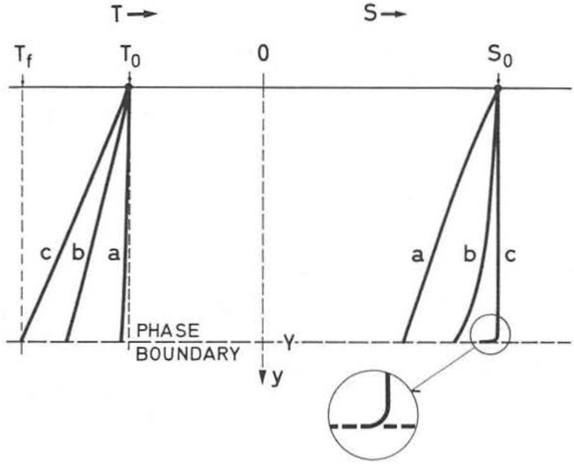

Figure 2 Temperature and salinity behaviour in the thawed layer: a for a purely diffusive regime, and b and c respectively for partially and strongly developed convective regimes.

Which of these transport regimes exists for given material properties and boundary conditions depends upon the state of development of the thawed layer thickness. When it is very thin H and O (1978) have shown that a purely diffusive regime is stable, and have calculated the thaw rate. But the same theory shows that soon, after a few years of submersion when the thawed layer is still only a fraction of a meter thick, convection should begin. At this point probably both the thaw rate and the strength of convection increase rapidly, and the strongly developed convective regime probably exists while the thawed layer is still thin. At this stage, as already described, the phase boundary temperature is very near the freezing temperature of the overlying water, and is therefore determined. Since $Pe_T \ll 1$, this means that subsequent development of the thawed layer can be calculated by heat conduction theory without further reference to the problems of salt transport. For example, if the initial temperature was roughly constant and the seabed temperature and salinity remain fixed, simple Stefan theory shows that the thawed layer thickness develops as $\sqrt{t}$. A scheme for calculating a more accurate time dependence is given by Lachenbruch and others (1977), but $\sqrt{t}$ is probably good enough for a simple model. It should be emphasized that transient effects associated with complex near shore conditions are probably much more important, and less understood, than any that might be assciated with an initially diffusive regime.

Despite some of the compilations discussed earlier, there seems to be no doubt that the salt convective regime at the West Dock is strongly developed. The phase boundary temperature is known and seems constant in time, and the sea bed temperature is known, so there is no serious problem in calculating temperature and thawed layer thickness. This means that a practical predictive tool exists. It also means that temperature is almost completely decoupled, in the sense that the salinity, velocity and pressure fields are governed by

$$\nabla \cdot \underset{\sim}{v} = 0,$$

$$v = -k(\nabla P' - a\, S'\, \hat{e}_g),$$

$$\kappa_S\, \nabla^2 S' - \underset{\sim}{v} \cdot \nabla S' - \frac{\partial S'}{\partial t} = 0,$$

subject to

$$S\Big|_0 = S_0 ,$$
$$v_x\Big|_0 = 0 ,$$
$$v_y\Big|_Y = 0 ,$$
$$\frac{\partial S}{\partial y}\Big|_Y = -\frac{S_Y}{\kappa_S}\frac{dY}{dt} \approx -\frac{S_0}{\kappa_S}\frac{dY}{dt} .$$

Temperature enters only indirectly, in determining the phase boundary position (Y), the depth at which the last two boundary conditions are to be applied. Unlike the conventional Stefan problem, this depth is known from the heat conduction calculation; neither it nor the salinity there has to be determined in the course of the solution of the above equations. Since this depth behaviour is known, dY/dt in the fourth boundary condition is known also, so the problem is completely specified.

APPLICATION OF THE MODEL

As a start, we should consider only those problems to which the simple decoupled model is applicable. Although the equations describing it may look simple, and can perhaps be attacked by boundary layer methods, their solution will be sufficiently difficult without the coupling with temperature and the Stefan boundary conditions of the full theory. Use of the decoupled model probably excludes application to the transient near shore region and to the effect of seasonal freezing at the sea bed, at least for now until we know more about the appropriate boundary conditions. Strictly speaking, it would seem to exclude the use of space and time-dependent boundary conditions in general, because then the assumption $S_0 \approx S_Y$ may be invalid. However, the field observation that S_Y is quite constant in time indicates that this point of view is too pessimistic. For example, one could test various sea bed salinity functions (S_0) to see if they produce the field observed essentially constant value of S_Y. Such an approach might also produce the high, slightly stratified salinity in the thawed layer that was described earlier. In the process, the time response of the system might become better understood.

However, the simplest way to begin is with a constant sea bed salinity. The phase boundary can be taken horizontal as well. These conditions do not imply that the convection pattern becomes two-dimensional, in fact it probably does not, but we could assume uniformity in the third dimension for a start. Even then, the motion of the lower boundary may make the problem too hard. It is possible that with the simple time dependence $Y \propto \sqrt{t}$ the equations can be scaled in a simple way to take account of the motion. However, it may be better to assume the boundary fixed, and to investigate the adequacy of the approximation a posteriori. For example, if a characteristic pore water velocity (v_c) is considerably greater than the phase boundary velocity (dY/dt), the approximation may be justified. Qualitative arguments in the next section suggest that this may be the case. It is very important to determine whether the fixed boundary approximation is valid. If so, it causes a final decoupling of the effect of temperature as far as the P', $\underset{\sim}{v}$ and S fields are concerned, in that a moving boundary would no longer have to be considered, even one that moved in a specified way. Of course Y and dY/dt would appear, but as far as the convection solution is concerned, only as constants.

It should be noted again that the porosity (ϕ), salt diffusivity (κ_s) and the quotient of hydraulic conductivity and porosity (k) are assumed constant, and in the case of κ_s and k, scalar as well. This is a fairly severe restriction. For example, the measured hydraulic conductivity varies by a factor of 3 or more about its mean value, although there may be slightly less variation in k, which is the quotient of hydraulic conductivity and porosity, and the parameter that occurs in the theory. At any rate, it is obvious that we do not expect to reproduce the details of the real velocity field, even if we apply the correct boundary conditions. However, we do expect to learn something about its general structure, the structure of the boundary layers, and what determines its time response. We are encouraged in these aims by the regularities that have been observed, especially by the $\sqrt{t}$ dependence of the thawed layer thickness after the near shore transient has died out, and by the uniform phase boundary temperature and salinity. We recall that the latter implies that this temperature and salinity have also been constant in time, because the spatial and temporal pictures are similar in a constant shore line retreat model.

SOME ASPECTS OF THE CONVECTIVE REGIME

We now consider some aspects of the convective regime that can be understood without formal solution of the model equations, requiring instead only field data or a combination of field data and model formulation. Numerical values are taken from Table 1.

The Peclet numbers

The Peclet numbers for salt (Pe_S) and for heat (Pe_T) are first considered. Their orders of magnitude have been known for some time (H & O, 1978; H & O, 1982) but a more systematic approach is possible. We define Pe_S by first specifying a "flux velocity" (v_F) as follows:

$$\text{Salt flux} \equiv v_F \, S_Y = S_Y \frac{dY}{dt} .$$

Therefore
$$v_F = \frac{dY}{dt}$$
$$\approx 0.028 \text{ m yr}^{-1} \text{ (700 m from shore)}.$$

Pe_S, defined as $\frac{v_F \, Y}{\kappa_S}$, is then

$$Pe_S = \frac{Y \frac{dY}{dt}}{\kappa_S} \approx 164 .$$

If we define a heat Peclet number (Pe_T) in terms of the same velocity,

$$Pe_T = \frac{Y \frac{dY}{dt}}{n} \approx \frac{1}{32} .$$

These results support the earlier statements that $Pe_S \gg 1$ and $Pe_T \ll 1$. These numerical values do not apply during the near-shore transient, but once it is over, and $Y \propto \sqrt{t}$, $Y \frac{dY}{dt}$ is constant. Therefore the Peclet numbers are also, independent of the subsequent thickness of the thawed layer.

The Rayleigh number for salt transport

If the salinity at the sea bed is fixed, and the motion of the phase boundary is ignored, a Rayleigh number (Ra) can be defined in the usual way:

$$Ra = \frac{k\ a\ Y\ s}{\kappa_S} ,$$

where s is some salinity difference. Since the salinity gradient is specified at the lower boundary, the appropriate s is what would be necessary to transport the flux thus specified by diffusion (Nield, 1968):

$$s = - \left. \frac{\partial S}{\partial y} \right|_Y Y$$

Eliminating $\left. \frac{\partial S}{\partial y} \right|_Y$ with the Stefan boundary condition, we get

$$Ra = \frac{k\ a\ S_Y\ Y^2 \frac{dY}{dt}}{{\kappa_S}^2}$$

$$\approx 342\,000 \text{ (700 m from shore)}.$$

Since convection is known to be strongly developed, such a high Rayleigh number is no surprise. Unlike the Peclet number, the Rayleigh number is not constant, but increases with thawed layer growth, becoming proportional to Y (since $Y \frac{dY}{dt}$ is constant) after the near shore transient is over.

Characteristic velocity

We next consider a "characteristic" velocity (v_C), which is not the same as the flux velocity v_F (H & O, 1982). If upon reaching the phase boundary the pore water gave up all its salt and then disappeared, v_C and v_F would be the same. However, in fact it gives up very little of its salt and then flows away again. The small salt release is characterized by the small jump ΔS across the boundary layer, which implies a salt release on the order of only 2 %. This suggests that v_C be defined by

$$\text{Salt flux} \equiv v_C \; \Delta S = S_Y \frac{dY}{dt}$$

so

$$v_C = \frac{S_Y}{\Delta S} \frac{dY}{dt} = \frac{S_Y}{\Delta S} v_F$$

$$\approx 1.2 \text{ m yr}^{-1} \text{ (700 m from shore)}$$

Because of the large uncertainity and probable variability in ΔS, this estimate for v_C is crude. We think of v_C as the order of magnitude that would be expected of a measurement. However, here is where a more complete solution of the model might help the interpretation. For example, is the flow concentrated in thin boundary layers between convecting cells? If so, the velocity there could be considerably greater than the above estimate.

We see that v_C is larger than dY/dt by the factor ($S_Y/\Delta S$), which we know is large. This suggests that the approximation of neglecting the phase boundary motion when solving the model equations may be valid.

Size scale of convection cells

Information about the size of the convection cells can be extracted using the observation that the phase boundary salinity S_Y is almost constant. In Figure 3 the bottom of a cell of size L is shown in contact with the phase boundary. The rate of salt supply to the boundary layer is $v\, S_Y\, b$, where b is the boundary layer thickness, and v is the velocity along

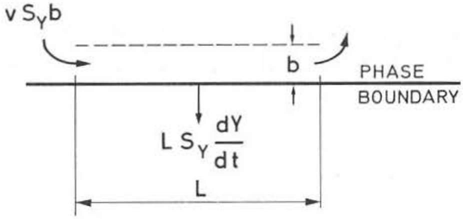

Figure 3 Contact between bottom of a convection cell of size L and phase boundary.

the phase boundary. The rate of loss of salt out of this layer to the thawing phase boundary is $L\ S_Y\ dY/dt$. Since S_Y is almost constant, this loss must be much less than the supply, or

$$v\ S_Y\ b >> L\ S_Y \frac{dY}{dt},$$

otherwise the salt concentration S_Y in the boundary layer would decrease from left ro right in the Figure. The factor b enters because only the salt within the boundary layer is available for thawing, since it has to be transported to the thaw front by diffusion. The boundary layer thickness (b) is defined by the Stefan condition

$$S_Y \frac{dY}{dt} = - \kappa_S \left.\frac{\partial S}{\partial y}\right|_Y \equiv \kappa_S \frac{\Delta S}{b}.$$

It seems reasonable to go a step further and note that the incoming salt supply is probably depleated by a factor of order $\Delta S/S_Y$ as it traverses the bottom of the cell. The inequality then becomes the order of magnitude equality

$$\frac{\Delta S}{S}\ v\ S_Y\ b \sim L\ S_Y \frac{dY}{dt}$$

Elimination of b and rearrangement give

$$v \sim \frac{L}{\kappa_S} \left(\frac{dY}{dt}\right)^2 \left(\frac{S_Y}{\Delta S}\right)^2.$$

If L is the same order of magnitude as Y, this leads to an unreasonably high v. For example, 700 m off shore $Y = 25$ m, which gives $v \sim 9000$ m yr^{-1}. A velocity of this magnitude over a distance of 25 m would require a head drop of order $\frac{v}{k}$ Y by Darcy's law. This gives 22 500 m, which is clearly impossible.

This seems to indicate that the cell size is much smaller than Y, at least near the phase boundary. In other words, the convection pattern may be complex, if an identifiable, approximately time-independent pattern exists at all. It is probably naive to think of a single, approximately equidimensional cell filling the entire thawed layer thickness.

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TABLE 1

$k \approx 10\ \mathrm{m\ yr^{-1}}$

$\kappa_S \approx 4\cdot 10^{-3}\ \mathrm{m^2\ yr^{-1}}$

$\kappa \approx 21\ \mathrm{m^2\ yr^{-1}}$

$a = 0.773$

$S_Y = 43\cdot 10^{-3}$ (43 ‰)

$Y = 25$ m (700 m from shore)

$\Delta S \approx 1\cdot 10^{-3}$ (average of values from Table 1, H and O, 1982)

$Y(\mathrm{m}) \approx 1.147\sqrt{t(\mathrm{yr})-276}$, $t > 440$ yr
(for a shore line retreat of $1\ \mathrm{m\,yr^{-1}}$)

GLACIER FLOW - RECENT DEVELOPMENTS

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ABSTRACT

Some new developments in glaciology are explained which are likely to have impact in future glaciological research.

ZUSAMMENFASSUNG

Einige neuere Entwicklungen in der Glaziologie werden dargelegt, die für die zukünftige Gletscherforschung von Bedeutung werden könnten.

GLACIER FLOW - RECENT DEVELOPMENTS

The motion of glaciers and ice sheets has fascinated mankind for more than a century; to some extent it still remains a mistery because the varied velocity scales are not jet fully understood. It appears that several mechanisms may be responsible for very similar flow conditions.

In general, large ice masses are polythermal that is, they consist of two zones, a cold zone, in which the temperature of the ice is below the melting point, and a temperate zone, in which the ice is at the melting point. In the cold zone, heat generated by internal friction affects the temperature distribution, and this in turn affects the motion. In the temperate zone, on the other hand, heat generated by friction melts some of the ice. Cold zones can be described in terms of a heat conducting viscous model, but temperate zones can be described only in terms of a model in which ice and water are treated in terms of a mixture of interacting ice and water, K. Hutter (1982a).

The flow and temperature distribution in glaciers depends on what happens within the ice mass and its bounding surfaces, the free surface, the ice-rock interface and the ice-water interface of possible floating portions. Cold and temperate ice are separated by an interface at which the temperature gradient is generally discontinuous. The body flow is governed by differential shearing; the value of it depends on the creep law of ice under slow plastic deformation. Older models relate the deformation rate to the stress with a stress dependent coefficient in such a way that in three dimensions isotropy of the ice is obtained, in partial disagreement with field observation. More realistic models of ice creep under plastic deformation orient themselves therefore at the anisotropy properties of the ice (L.W. Morland and U. Spring 1982).

The overall velocities in a glacier depend crucially on what happens at the ice-rock interface. The boundary conditions depend on the fact whether the ice is cold or temperate. At the cold portions of the base the no-slip condition applies; adhesive forces prevent the ice from sliding over the bedrock. When the ice at the base is temperate, however, the heat generated by deformation causes melting; hence a layer of water separates the bottom from the rock and acts as a lubricant thereby allowing the ice to slide. The thickness of the lubricating film may be negligibly small, but it is sufficient to change the boundary condition from no-slip to perfect sliding. Bed resistance is now only due to pressure variations of the undulating bed. On occasion, in Alpine regions when summer meltwater becomes abundant, interstitial water pressure is large and may partly lift the ice from its bedrock. Cavities are formed, the effective bed is thereby smoothed and the effective sliding resistivity therefore reduced. The functional relationship connecting the basal sliding velocity u_b and the basal shear traction τ_b is qualitatively known for sliding without cavity formation and as shown in Figure 1a, J Weertman (1979). When cavities are formed Lliboutry (1979) gives arguments which suggest behaviors as shown in Figures 1b or 1c. Here, to a given basal traction two or more values of the sliding velocity exist, giving rise to a bifurcation mechanism as explained in the Figure caption. The transition from the situation of Figure 1a to that of 1c is described by a variable connected to the water content in the basal cavities. It is believed that this variable is the interstitial pressure. A. Iken (1981) and others (1982) have collected field evidence in support of this. A more complete picture of basal sliding is then probably as displayed and explained in Figure 2.

The multiplicity of flow conditions suggested by Figure 2 is physically impossible. Under such circumstances, nature favors the stable configuration. Hence, if a glacier is in another configuration, which is unstable, any perturbation

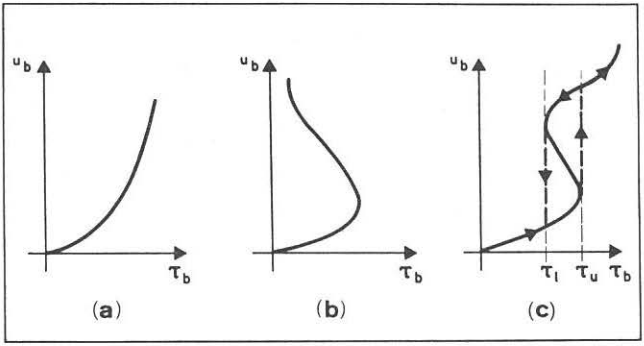

Figure 1 There are three possible relationships between the component of the velocity parallel to the smoothed-out base, u_b, and the shear traction, τ_b. In the first, a, which represents sliding without cavity formation, u_b and τ_b are uniquely related. The second and third, b and c, which are speculative but are thought to be representative of sliding with cavity formation, allow for the flow bifurcation occasionally observed in glacier surges, and indeed, the third case, c, may explain surging motion. The S-shaped curve (color) is replaced by a sledge-type curve (black). On the lower part of the curve, u_b increases slowly with τ_b until an upper limit, τ_u, is reached; u_b then jumps suddenly to a much higher value. Conversely, when starting at high values of τ_b, u_b decreases slowly with decreasing τ_b until a lower limit, τ_1, is reached, at which point it suddenly drops to a much lower value, which corresponds to a sudden slowing of the ice.
(From American Scientist, Vol. 70, Jan./Feb. 1982, pp. 26-34. Article by K. Hutter: Glacier Flow.)

will cause it to bifurcate rapidly into the stable configuration. Such a bifurcation may successfully explain surges. Suring glaciers move over a relatively short period of time - 1 to 2 years - at velocities several orders of magnitude greater than those of normal glaciers. One cause of the surges is the peculiar sliding law of Figure 2. There are, however, also others, in particular because also some cold glaciers are known to occasionally surge.

In cold ice the velocity and the temperature distribution

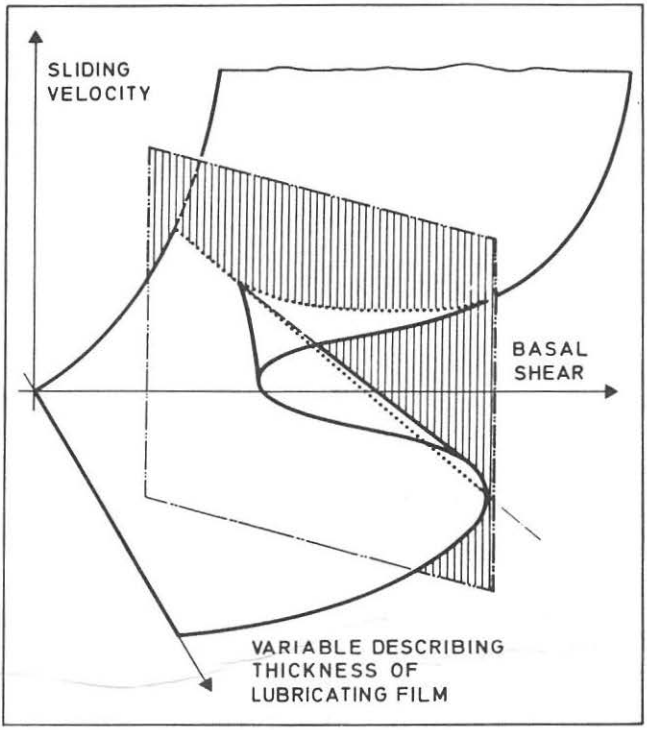

Figure 2 It has been suggested that the transition from the unique relationship describing sliding without cavity formation to the relationship describing sliding with cavity formation can be understood in terms of a curtain-folded surface. The curves cut out of this surface by planes parallel to the sliding velocity - basal shear stress are of the forms shown in a and c of Figure 1. (From American Scientist, Vol. 70, Jan./Feb. 1982, pp. 26-34. Article by K. Hutter: Glacier Flow.)

interact in a non-linear fashion. G.C.K. Clarke, U. Nitson and W.S.B. Paterson (1977) have shown, using a simplified mathematical mode, that under certain conditions of geothermal heat flux and atmospheric temperature the longitudinal surface velocity is non-uniquely related to these thermal boundary conditions. In the multivalued range, to a given geothermal heat flow there exist a small and a large streamwise surface velocity, providing another possible bifurcating mechanism, but it is not firmly known yet whether the bifurcation would correspond to a transition from an unstable to a stable flow configuration.

Prediction of ice sheet profile geometry is one of the most intriguing questions of glacio-climatology, because it will ultimately answer the still unknown question of disintegration and formation of large ice sheets and will allow prediction of glacier advance and retreat on a more local scale. First and simple models fail frequently to match observational facts and bear the disadvantage of not being set on a rigorous footing of basic physical laws. When attempting to predict mathematically the surface profile of an ice mass under steady or transient conditions the profile geometry must be determined together with the associated flow and temperature fields. By incorporating into the governing equations the fact that ice sheets are long and wide in comparison to their maximum thickness L.W. Morland & H. Johnson (1982) and Hutter (1982b) succeeded in describing a workable scheme resulting in a non-linear set of equations for the surface profile that incorporates differential creep and basal sliding, yet allows the accumulation versus ablation rate function and the geothermal heat to be freely and realistically assigned.

In applied glaciology localized flow problems are generally in the foreground; it is indeed so that glaciers are often being observed, with a view to preventing catastrophs involving human beings. Ice avalanches need not be catastrophes, because they are often preceeded by unusually large surface velocities, which increase in time and can therefore be measured and used to forecast avalanches. Ice with such atypically high velocities is confined to limited regions, and the date of the catastrophe can be forecasted by fitting a hyperbola through the velocity versus-time curve of a typical point and identifying the catastrophe with the asymptote of the hyperbola.

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